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Ivane Javakhishvili
Tbilisi State University

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BOOK OF ABSTRACTS

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2. Operator Theory Methods in Singular Integral Equations (OTMSIE)
3. Variational Methods and Applications
4. Toeplitz operators and related topics
5. Algebraic and analytic aspects of Hilbert space operators
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7. Operator Theory, Real Algebraic Geometry, And Moment Problems
8. Free Noncommutative Analysis And Its Applications
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Abstracts of Plenary Talks

Multivariable Nevanlinna–Pick Interpolation: the Free Noncommutative Setting

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We are now approaching the centennial of the discovery of Nevanlinna (1919) and Pick (1916) of what is now called **Nevanlinna–Pick interpolation**: *given a finite collection of points* $z_1, \ldots, z_N$ *in the unit disk* $\mathbb{D}$ *and a corresponding finite collection of prescribed values* $w_1, \ldots, w_N$ *in* $\mathbb{C}$*, then there is a holomorphic function* f *mapping the unit disk* $\mathbb{D}$ *into the closed disk* $\overline{\mathbb{D}}$ *satisfying the prescribed interpolation conditions* $f(z_i) = w_i$ *if and only if the associated Pick matrix*

$$\mathbb{P} = \left[\frac{1 - \lambda_i \overline{\lambda_j}}{1 - z_i \overline{z_j}}\right]$$

is positive semidefinite. Over the years there has now been a lot of developments extending the theory of Nevanlinna–Pick interpolation to more general target spaces (matrix- or operator-valued instead of scalar-valued—significantly inspired by connections with the emerging H^∞-control theory) as well as to more general domains (more general planar as well as multivariable domains in place of the unit disk). Traditionally these multivariable domains involve only commuting variables.

It is only recently that a new "free analysis", or the study of "noncommutative functions" has emerged, where the domain consists of freely noncommuting variables into which one plugs matrix- or even operator-valued arguments. A **noncommutative function** is defined to be a function of freely noncommuting square-matrix arguments (of arbitrary size) which is required to have certain natural invariance properties with respect to direct sums and similarity transforms; these axioms combined with weak boundedness assumptions lead to strong analyticity properties. A number of groups of researchers have come upon this class of functions for completely independent reasons (development of a meaningful functional calculus for noncommuting operator tuples, the theory of formal languages and free automata, free probability, dimension-less optimization problems in systems engineering, input/state/output linear systems with evolution along a free monoid). We refer to the recent monograph [1] for more complete details on both the historical context and the basic theory.

In this talk we discuss a free version of the Nevanlinna–Pick interpolation problem: *given prescribed interpolation nodes* $Z^{(1)}, \ldots, Z^{(N)}$ *in a noncommutative domain* $\mathbb{D}_Q$ *and associated matrix values* $\Lambda_1, \ldots, \Lambda_N$*, find a contractive noncommutative function* S *on* $\mathbb{D}_Q$ *so that* $S(Z^{(i)}) = \Lambda_i$ *for* $i = 1, \ldots, N$. We emphasize both the similarities with and the points of departure from the earlier (commutative-variable) versions of Nevanlinna–Pick interpolation. This is joint work with Gregory Marx (Virginia Tech) and Victor Vinnikov (Ben Gurion University).

References

[1] D. S. Kaliuzhnyi-Verbovetskyi and V. Vinnikov, Foundations of free noncommutative function theory. Mathematical Surveys and Monographs, 199. *American Mathematical Society, Providence, RI*, 2014.

Toeplitz Determinants and Lattice Theory

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A lattice is a discrete subgroup in a finite-dimensional Euclidean space. Lattice theory has numerous applications, for instance, in discrete optimization or coding theory. One of these applications consists in associating lattices with elliptic or related curves over finite fields, which are of prominent use in coding theory, and then to connect arithmetic properties of the curve with geometric properties of the lattice. A basic quantity of every lattice is the volume of its fundamental domains. This volume is the determinant of some matrix, and in several interesting cases this matrix is just a perturbed Toeplitz matrix. The generating function of this Toeplitz matrix is in general not well-behaved, so that the classical Szegő limit theorem for Toeplitz determinants cannot be used. It rather turns out that the generating function is often a so-called Fisher–Hartwig symbol. As such symbols are also emerging in statistical physics, their determinants have been thoroughly studied for decades. The results of these studies now prove to be of use in lattice theory. The talk is an introduction to some aspects of lattice theory and Toeplitz determinants and also exhibits some recent results obtained in joint work with Lenny Fukshansky, Stephan Ramon Garcia, and Hiren Maharaj in [1, 2].

References

[1] A. Böttcher, L. Fukshansky, S. R. Garcia, and H. Maharaj, Toeplitz determinants with perturbations in the corners. *J. Funct. Anal.* **268** (2015), No. 1, 171–193.

[2] A. Böttcher, L. Fukshansky, S. R. Garcia, and H. Maharaj, On lattices generated by finite Abelian groups. *SIAM J. Discrete Math.* **29** (2015), No. 1, 382–404.

Spectral Estimates on Manifolds

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In this talk I will present recent work with Jean Dolbeault, Ari Laptev and Michael Loss about optimal estimates for the principal eigenvalue of Schrdinger operators on compact and non compact manifolds based on the best constants for some functional inequalities. These estimates show that for non-flat manifolds both Keller and Lieb-Thirring-like estimates do not hold true with the usual constants and exponents as in the Eucledian space.

Limit Spectrum Graph for Some Non-Self-Adjoint Differential Operators with Small Physical Parameter

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The celebrated Orr–Sommerfeld equation

$$\Big\{(D^2-\alpha^2)^2 - i\alpha R\big[q(x)(D^2-\alpha^2) - q''(x)\big]\Big\}y = -i\alpha R\lambda(D^2-\alpha^2)y,$$

comes from the linearization of the Navie–Stokes equation in the infinite 3-dim spacial layer $|x| \leqslant 1$, $\xi, \eta \in \mathbb{R}^2$. Here R is the Reynolds number (which characterizes the viscosity of the liquid), α is the wave number, $q(x)$ is the velocity profile, λ is the spectral parameter, and $D = d/dx$. It is the old problem in hydrodynamics to understand the spectrum behavior of the Orr–Sommerfeld equation as $R \to \infty$ (when the liquid becomes close to the ideal one). It turns out that the simpler model problem

$$-i\varepsilon y'' + q(x)y = \lambda y, \quad y(\pm 1) = 0, \quad \varepsilon = 1/R \to 0,$$

is closely related to the above one.

There is a great amount of works devoted to both problems. However, there are not too many rigorous results on this topic. The spectrum behavior of the above problems depends dramatically on the analytical properties of the profile q. Our aim is to describe the spectral portraits of the model problem as $\varepsilon \to 0$ for the case when q is a polynomial. It turns out that in this case the spectrum concentrates along some critical curves in the complex plane which form "the limit spectral graph". The problem is to understand the geometry of this graph, to find analytic formulae for the curves describing its parts and to write the asymptotic eigenvalue distribution (uniformly, as $\varepsilon \to 0$) along the limit spectral curves. All these problems will be discussed in the talk and the obtained results will be formulated.

The talk is based on the joint works of the author with S. N. Tumanov.

Abstracts of Invited Talks

Spectral Regularity of Banach Algebras and Sums of Idempotents

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A basic result from complex function theory states that the contour integral of the logarithmic derivative of a scalar analytic function can only vanish when the function has no zeros inside the contour. Question: *does this result generalize to the vector-valued case?*

We suppose the function takes values in a Banach algebra. The answer depends on which Banach algebra. Positive results have been obtained for large classes of algebras, among them that of the polynomial identity Banach algebras. To deal with the latter, one needs non-commutative Gelfand theory which involves the use of families of matrix representations.

Logarithmic residues have much to do with sums of idempotents. Pursuing this connection, negative answers to the above question have come up via the construction of non-trivial zero sums of a finite number of idempotents. It is intriguing that we need only five idempotents in all known examples. The idempotent constructions relate to deep problems concerning the geometry of Banach spaces and general topology. In particular a novel approach to the construction of Cantor type sets plays a role.

The talk is a report on joint work with Torsten Ehrhardt (Santa Cruz, California) and Bernd Silbermann (Chemnitz, Germany).

Non-Extremal Sextic Moment Problems

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For a degree $2n$ complex sequence $\gamma \equiv \gamma^{(2n)} = \{\gamma_{ij}\}_{i,j\in Z_+,\, i+j\leqslant 2n}$ to have a representing measure μ, it is necessary for the associated moment matrix $M(n)$ to be positive semidefinite, and for the algebraic variety associated to γ, $\mathscr{V}_\gamma \equiv \mathscr{V}(M(n))$, to satisfy $\operatorname{rank} M(n) \leqslant \operatorname{card} \mathscr{V}_\gamma$ as well as the following *consistency* condition: if a polynomial $p(z,\overline{z}) \equiv \sum_{ij} a_{ij}\overline{z}^i z^j$ of degree at most $2n$ vanishes on $\mathscr{V}_\gamma$, then the *Riesz functional* $\Lambda(p) \equiv p(\gamma) := \sum_{ij} a_{ij}\gamma_{ij} = 0$.

Positive semidefiniteness, recursiveness, and the variety condition of a moment matrix are necessary and sufficient conditions to solve the quadratic ($n = 1$) and quartic ($n = 2$) moment problems. Also, positive semidefiniteness, combined with consistency, is sufficient in the case of *extremal* moment problems, i.e., when the rank of the moment matrix (denoted by r) and the cardinality of the associated algebraic variety (denoted by v) are equal. However, these conditions are not sufficient for *non*-extremal (i.e., $r < v$) sextic ($n = 3$) or higher-order truncated moment problems.

Let $n = 3$, assume that $M(3) \geqslant 0$, and that it satisfies the variety condition $\operatorname{rank} M(3) \leqslant \operatorname{card} \mathscr{V}_\gamma$ as well as consistency. Also assume that $M(3)$ admits at least one *cubic* column relation. We prove the existence of a related matrix $\widetilde{M(3)}$ with $\operatorname{rank} \widetilde{M(3)} < \operatorname{rank} M(3)$ and such that each representing measure for $\widetilde{M(3)}$ gives rise to a representing measure for $M(3)$. As a concrete application, we discuss the case when $\operatorname{rank} M(3) = 8$ and $\operatorname{card} \mathscr{V}(M(3)) \leqslant 9$.

Along the way, we settle three key instances of the non-extremal sextic moment problem, as follows: when $r = 7$, positive semidefiniteness, consistency and the variety condition guarantee the existence of a 7-atomic representing measure; when $r = 8$ we construct two determining algorithms, corresponding to the cases $v = 9$ and $v = +\infty$. To accomplish this, we generalize the above mentioned rank-reduction technique, which was used in previous work to find an explicit solution of the nonsingular quartic moment problem.

The talk is based on joint work with Seonguk Yoo.

Skew-Selfadjoint Dirac Systems with a Rational Weyl Function: Direct and Inverse Problems, and Related Nonlinear Equations

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In this talk we deal with skew-selfadjoint differential systems of Dirac type, continuous or discrete time, and in both cases the Weyl function is assumed to be a rational matrix function. This allows one to use state space techniques from mathematical system and control theory to solve explicitly various direct and inverse problems, and to obtain explicit solutions for a number of classical non-linear PDEs. The latter appear when the corresponding pseudo-canonical potential depends on an additional second variable. We shall review some of the earlier results for the continuous time case derived in joint work with the late Israel Gohberg and Alexander Sakhnovich [1]. In the second part of the talk the emphasis will be on the discrete time setting. In this case, using state space techniques, explicit solutions are obtained for the so-called generalized discrete Heisenberg magnet model. This part of the talk is based on recent joint work [2, 3] with Bernd Fritzsche, Bernd Kirstein, and Alexander Sakhnovich.

References

[1] I. Gohberg, M. A. Kaashoek, and A. L. Sakhnovich, Pseudo-canonical systems with rational Weyl functions: explicit formulas and applications. *J. Differential Equations* **146** (1998), No. 2, 375–398.

[2] M. A. Kaashoek and A. L. Sakhnovich, Discrete skew self-adjoint canonical system and the isotropic Heisenberg magnet model. *J. Funct. Anal.* **228** (2005), No. 1, 207–233.

[3] B. Fritzsche, M. A. Kaashoek, B. Kirstein, and A. L. Sakhnovich, Skew-selfadjoint Dirac systems with rational rectangular Weyl functions: explicit solutions of direct and inverse problems and integrable wave equations. `http://arxiv.org/abs/1501.00395`.

Wave Diffraction by Wedges Having Arbitrary Aperture Angle

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The problem of plane wave diffraction by a wedge sector having arbitrary aperture angle has a very long and interesting research background. In fact, we may recognize significant research on this topic for more than one century. Despite this fact, up to now no clear unified approach was implemented to treat such a problem from a rigourous mathematical way and in a consequent appropriate Sobolev space setting. In the present paper, we are considering the corresponding boundary value problems for the Helmholtz equation, with complex wave number, admitting combinations of Dirichlet and Neumann boundary conditions. The main ideas are based on a convenient combination of potential representation formulas associated with (weighted) Mellin pseudo-differential operators in appropriate Sobolev spaces, and a detailed Fredholm analysis. Thus, we prove that the problems have unique solutions (with continuous dependence on the data), which are represented by the single and double layer potentials, where the densities are solutions of derived pseudo-differential equations on the half-line.

The talk is based on joint work with co-author Luís P. Castro from the University of Aveiro.

Commuting Dilations and Linear Matrix Inequalities

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Given a tuple $A = (A_1, \dots, A_g)$ of real symmetric matrices of the same size, the affine linear matrix polynomial $L(x) := I - \sum A_j x_j$ is a monic linear pencil. The solution set S_L of the corresponding linear matrix inequality, consisting of those x in R^g for which $L(x)$ is positive semidefinite (PsD), is a spectrahedron. It is a convex semialgebraic subset of R^g. The set D_L of tuples $X = (X_1, \dots, X_g)$ of symmetric matrices (of the same size) for which $L(X) := I - \sum A_j \otimes X_j$ is PsD, is called a free spectrahedron. We explain that any tuple X of symmetric matrices in a bounded free spectrahedron D_L dilates, up to a scale factor, to a tuple T of commuting self-adjoint operators with joint spectrum in the corresponding spectrahedron S_L. The scale factor measures the extent that a positive map can fail to be completely positive. In the case when S_L is the hypercube $[-1, 1]^g$, we derive an analytical formula for this scale factor, which as a by-product gives new probabilistic results for the binomial and beta distributions.

This is based on joint work with Bill Helton, Scott McCullough and Markus Schweighofer.

Global Spatial Regularity Results for Elasticity Models with Cracks, Damage, Contact and Other Nonsmooth Constraints

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For the analysis of strongly coupled material models it is useful to have deeper insight into the spatial regularity properties of the involved quantities like displacement fields or internal variables. In this lecture we will discuss some recent results for non-smooth situations with a special focus contact problems with nonsmooth obstacles, friction models and for certain time-dependent damage models in the small strain regime.

The talk relies on joint results with R. Rossi (University of Brescia), C. Zanini (Politecnico di Torino) and A. Schröder (University of Salzburg).

References

[1] D. Knees and A. Schröder, Global spatial regularity for elasticity models with cracks, contact and other nonsmooth constraints. *Math. Methods Appl. Sci.* **35** (2012), No. 15, 1859–1884.

[2] D. Knees, R. Rossi, and Ch. Zanini, A quasilinear differential inclusion for viscous and rate-independent damage systems in non-smooth domains. *Nonlinear Anal. Real World Appl.* **24** (2015), 126–162.

The String Density Problem and Nonlinear Wave Equations

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Classical objects in spectral theory are the differential equation

$$-y'' = z\,\omega\,y, \quad x \in [0, L), \tag{1}$$

(here $L \in (0, \infty]$, ω is a positive Borel measure on $[0, L)$ and z is a spectral parameter) and the Weyl–Titchmarsh m-function, which encodes all the spectral information about (1). In a series of papers in the 1950s, M. G. Krein investigated the direct and inverse spectral problems for this equation. Viewing these problems as a natural generalization of investigations of T. Stieltjes on continued fractions to the moment problem, M. G. Krein identified the totality of all possible m-functions with the class of the so-called Stieltjes functions in a bijective way. Recently, the string density problem has come up in connection with some completely integrable nonlinear wave equations (e.g., the Camassa–Holm equation) for which the string spectral problem serves as an underlying isospectral problem. In contrast to the KdV equation, the Camassa–Holm equation possesses peaked solitons, called peakons, and models breaking waves. The latter happens when ω is a signed measure, i.e., the string is *indefinite*.

In this talk, we review the direct and inverse spectral theory for indefinite strings and relate it to the conservative Camassa–Holm flow. As one of our main results we are going to present the indefinite analog of M. G. Krein's celebrated solution to the string density problem. A special attention will be given to multi-peakon solutions.

The talk is based on joint work with Jonathan Eckhardt.

On Inverse Continuity of the Numerical Range Generating Function

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The *numerical range*, a.k.a. the *field of values*, or the *Hausdorff set* of a linear bounded operator A on a Hilbert space $\mathscr{H}$, is the range of the map $f_A(x) = \operatorname{scal} Ax, x$ acting on the unit sphere in $\mathscr{H}$. We consider the continuity properties of the (multivalued) inverse function f_A^{-1}, distinguishing between weak continuity, strong continuity, and existence of single-valued continuous selections. It is established in particular that strong continuity holds on the interior of $F(A)$, and that in finite dimensional setting it may fail only at finitely many points, which have to be round multiply generated boundary points.

The talk is based in part on publications [1–5]. Some applications of the results obtained there to quantum mechanics are in [6].

References

[1] D. Corey, Ch. R. Johnson, R. Kirk, B. Lins, and I. Spitkovsky, Continuity properties of vectors realizing points in the classical field of values. *Linear Multilinear Algebra* **61** (2013), No. 10, 1329–1338.

[2] T. Leake, B. Lins, and I. M. Spitkovsky, Pre-images of boundary points of the numerical range. *Oper. Matrices* **8** (2014), No. 3, 699–724.

[3] T. Leake, B. Lins, and I. M. Spitkovsky, Inverse continuity on the boundary of the numerical range. *Linear Multilinear Algebra* **62** (2014), No. 10, 1335–1345.

[4] B. Lins and P. Parihar, Continuous selections of the inverse numerical range map. *Linear Multilinear Algebra*, 2015 (to appear).

[5] T. Leake, B. Lins, and I. M. Spitkovsky: Corrections and additions to "inverse continuity on the boundary of the numerical range. *Linear Multilinear Algebra*, 2015 (to appear).

[6] L. Rodman, I. M. Spitkovsky, A. Szkoła, and S. Weis, Continuity of the maximum-entropy inference: Convex geometry and numerical ranges approach. `http://arxiv.org/abs/1502.02018`.

Fractional Integral Operators Between Banach Function Lattices

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We study the generalized fractional integral transforms associated to a measure on a quasi-metric space. We give a characterization of those measures for which these operators are bounded between L_p-spaces defined on nonhomogeneous spaces. We provide necessary and sufficient conditions for some classes of integral operators to be bounded from Lorentz to Marcinkiewicz spaces. The boundedness of multisublinear operators generated by quasi-concave functions between weighted Banach function lattices is established. These operators, in particular, generalize the Hardy–Littlewood and fractional maximal functions playing an important role in Harmonic Analysis. Under some general geometrical assumptions on Banach function lattices two-weight estimates for these operators are derived.

The talk is based on the recent papers [1, 2].

References

[1] V. Kokilashvili, M. Mastyło, and A. Meskhi, Fractional integral operators between Banach function lattices. *Nonlinear Anal.* **117** (2015), 148–158.

[2] V. Kokilashvili, M. Mastyło, and A. Meskhi, The multisublinear maximal type operators in Banach function lattices. *J. Math. Anal. Appl.* **421** (2015), No. 1, 656–668.

Functions of Noncommuting Self-Adjoint Operators Under Perturbation and Triple Operator Integrals

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We define functions $f(A, B)$ for noncommuting self-adjoint operators in terms of double operator integrals. We study the problem for which functions f Lipschitz type estimates

$$\big\|f(A_1, B_1) - f(A_2, B_2)\big\| \leqslant const \max\big\{\|A_1 - A_2\|, \|B_1 - B_2\|\big\}$$

hold in various norms (the operator norm, the Schatten von Neumann norms).

We prove that for functions f in the Besov space $B^1_{\infty,1}$ of two variables the above Lipschitz type estimate holds in the Schatten–von Neumann norm S_p for $p \in [1,2]$. However, it does not hold in the operator norm and it does not hold in the Schatten–von Neumann norm S_p for $p > 2$.

To obtain Lipschitz type estimate, we find a representation of $f(A_1, B_1) - f(A_2, B_2)$ in terms of triple operator integrals and we study Schatten–von Neumann properties of triple operator integrals.

In particular, we establish the following formula. Suppose that $\in B^1_{\infty,1}$, A_1, B_1, A_2, B_2 are self-adjoint operators such that both $A_1 - A_2$ and $B_1 - B_2$ belong to the Schatten–von Neumann class S_p with $p \in [1,2]$. Then the following formula holds:

$$\begin{aligned} f(A_1,B_1) - f(A_2,B_2) = \\ &= \iiint \frac{f(x_1,y) - f(x_2,y)}{x_1 - x_2}\, dE_{A_1}(x_1)(A_1 - A_2)\, dE_{A_2}(x_2)\, dE_{B_1}(y), \\ &+ \iiint \frac{f(x,y_1) - f(x,y_2)}{y_1 - y_2}\, dE_{A_2}(x)\, dE_{B_1}(y_1)(B_1 - B_2)\, dE_{B_2}(y_2), \end{aligned}$$

where E_{A_1}, E_{A_2}, E_{B_1}, E_{B_2} are the spectral measures of A_1, A_2, B_1 and B_2.

To deduce from this formula the above Lipschitz type inequality, we obtain new results on Schatten–von Neumann properties of triple operator integrals.

The results are based on my joint work with A. B. Aleksandrov and F. L. Nazarov.

Asymptotics of the Smallest Singular Value of a Class of Toeplitz Matrices and Related Rank One Perturbations

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Square matrices of the form $X_n = T_n + f_n(T_n^{-1})^*$, where T_n is an $n \times n$ invertible banded Toeplitz matrix and f_n some positive sequence are considered. The norms of their inverses are described asymptotically as their size n increases. As an example, for

$$X_n = \begin{bmatrix} 1+\frac{1}{n} & -1 & 0 & \cdots & \cdots & 0 \\ \frac{1}{n} & 1+\frac{1}{n} & -1 & 0 & & \vdots \\ \vdots & & & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & -1 & 0 \\ \vdots & \ddots & \ddots & \ddots & & -1 \\ \frac{1}{n} & \cdots & \cdots & \cdots & \frac{1}{n} & 1+\frac{1}{n} \end{bmatrix},$$

it will be shown that

$$\lim_{n\to\infty} \frac{2\|X_n^{-1}\|}{\sqrt{n}} = 1.$$

Certain finite rank perturbations of these matrices are shown to have no effect on this behaviour. In the concrete example above, for the matrix K_n obtained from X_n by adding one to each entry in the first column, one also has the same asymptotics for the norm of the inverse.

Finally, the singular vectors of a related Toeplitz matrix exhibit a pepeculiar permutation phenomenon. To explain this phenomenon, explicit formulas for the singular values and the singular vectors are given. Although these formulas are known (see [4]), the permutation phenomenon seems to have gone unnoticed thus far.

References

[1] H. Rabe and A. C. M. Ran, Asymptotics of the smallest singular value of a class of Toeplitz-generated matrices and related finite rank perturbations. *Integral Equations Operator Theory* **77** (2013), No. 3, 385–396.

[2] H. Rabe and A. C. M. Ran, Asymptotics of the smallest singular value of a class of Toeplitz-generated matrices II. *Integral Equations Operator Theory* **79** (2014), No. 2, 243–253.

[3] H. Rabe and A. C. M. Ran, A peculiar permutation phenomenon arising from the singular vector entries of a special class of Toeplitz matrices. *Linear Algebra Appl.* **459** (2014), 368–383.

[4] G. Strang, The discrete cosine transform. *SIAM Rev.* **41** (1999), No. 1, 135–147 (electronic).

Limit Operators and their Applications to Mathematical Physics

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The talk is devoted to applications of the limit operators method (see [1])to some problems of Mathematical Physics. We will consider:

- The essential spectra some discrete models of Mathematical Physics (see [2, 3]);
- The essential spectra and exponential estimates at infinity of eigenfunctions of general differential and pseudodifferential operators on $\mathbb{R}^n$, in particular, Schrödinger and Dirac operators with general potentials (see [4, 5]);
- Fredholm properties of pseudodifferential operators on some non compact surfaces in $\mathbb{R}^n$ with applications to the scattering problems on unbounded obstacles (see [6]).

References

[1] V. Rabinovich, S. Roch, and B. Silbermann, Limit operators and their applications in operator theory. Operator Theory: Advances and Applications, 150. *Birkhäuser Verlag, Basel*, 2004.

[2] V. S. Rabinovich and S. Roch, Essential spectra and exponential estimates of eigenfunctions of lattice operators of quantum mechanics. *J. Phys. A* **42** (2009), No. 38, 385207, 21 pp.

[3] V. S. Rabinovich and S. Roch, Essential spectra of difference operators on $\mathbb{Z}^n$-periodic graphs. *J. Phys. A* **40** (2007), No. 33, 10109–10128.

[4] V. S. Rabinovich, Essential spectrum of perturbed pseudodifferential operators. Applications to the Schrödinger, Klein-Gordon, and Dirac operators. *Russ. J. Math. Phys.* **12** (2005), No. 1, 62–80.

[5] V. S. Rabinovich and S. Roch, Essential spectrum and exponential decay estimates of solutions of elliptic systems of partial differential equations. Applications to Schrdinger and Dirac operators. *Georgian Math. J.* **15** (2008), No. 2, 333–351.

[6] V. Rabinovich, The Fredholm property and essential spectra of pseudodifferential operators on non-compact manifolds and limit operators. *Contemporary Mathematics, AMS*, 2015.

On the Definition of Pseudospectra

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Some authors define the epsilon-pseudospectrum of a linear operator with the help of a strict inequality, some others – with the help of a non-strict one. The difference between the resulting sets is a level set of the resolvent norm of the operator in question. In 1976, J. Globevnik posed a question on whether or not such a level set for a bounded linear operator can have a non-empty interior, i.e. whether or not the resolvent norm of a bounded linear operator on a Banach space can be constant on an open set. The question remained open until 2008.

The talk is a survey of results related to Globevnik's question, in particular of a recent joint work with E. B. Davies.

Operator Relations in Boundary Value Problems

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We study relations between operators which appear in the context of linear boundary value problems. Typically the operator associated to a BVP is "equivalently reduced" to a another "simpler" operator that, e.g., appears in boundary integral equations or in a corresponding semi-homogeneous problem etc. Some questions are: I. Which kind of relations appear? What are their properties? II. How can this knowledge be used to make the reasoning more transparent and efficient? III. What are the consequences concerning explicit solution, regularity, asymptotic results, normalization of ill-posed problems etc.?

Particular attention is devoted to problems which are not well-posed and even not Fredholm. Examples are given from diffraction theory, leading to Wiener-Hopf plus Hankel and Fourier integral operator matrices. In short: Strong relations (like the equivalence after extension relation) transfer nice properties such as an explicit computation of generalized inverses or allow conclusions such as the discovery of compatibility conditions for finding convenient space settings.

The talk is based upon joint work with L. P. Castro, A. Moura Santos and F. S. Teixeira.

References

[1] L. P. Castro and F.-O. Speck, Regularity properties and generalized inverses of delta-related operators. *Z. Anal. Anwendungen* **17** (1998), No. 3, 577–598.

[2] L. P. Castro, F.-O. Speck, and F. S. Teixeira, Mixed boundary value problems for the Helmholtz equation in a quadrant. *Integral Equations Operator Theory* **56** (2006), No. 1, 1–44.

[3] L. P. Castro, F.-O. Speck, and F. S. Teixeira, A direct approach to convolution type operators with symmetry. *Math. Nachr.* **269/270** (2004), 73–85.

[4] L. Castro, F.-O. Speck, and F.S. Teixeira, On a class of wedge diffraction problems posted by Erhard Meister. *Operator theoretical methods and applications to mathematical physics*, 213–240, Oper. Theory Adv. Appl., 147, *Birkhäuser, Basel*, 2004.

[5] A. Moura Santos, F.-O. Speck, and F. S. Teixeira, Minimal normalization of Wiener–Hopf operators in spaces of Bessel potentials. *J. Math. Anal. Appl.* **225** (1998), No. 2, 501–531.

[6] F.-O. Speck, On the reduction of linear systems related to boundary value problems. *Operator theory, pseudo-differential equations, and mathematical physics*, 391–406, Oper. Theory Adv. Appl., 228, *Birkhäuser/Springer Basel AG, Basel*, 2013.

[7] F.-O. Speck, Diffraction from a three-quarter-plane using an abstract Babinet principle. *ZAMM Z. Angew. Math. Mech.* **93** (2013), No. 6-7, 485–491.

Toeplitz Operators Defined by Sesquilinear Forms

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The classical theory of Toeplitz operators in spaces of analytic functions (Hardy, Bergman, Fock, etc spaces) deals usually with symbols that are bounded measurable functions on the domain in question. A further extension of the theory was made for symbols being unbounded functions, measures, and compactly supported distributions.

For a reproducing kernel Hilbert space we describe a certain common pattern, based on the language of sesquilinear forms, that permits us to introduce a further substantial extension of a class of admissible symbols that generate bounded Toeplitz operators.

Although the approach is unified for all reproducing kernel Hilbert spaces, for concrete operator consideration in this talk we restrict ourselves to Toeplitz operators acting on the standard Fock and Bergman spaces.

The talk is based on joint work with Grigori Rozenblum, Chalmers University of Technology, Gothenburg, Sweden.

Characteristic Functions, Systems, Discriminant Curves and Vessels: the Ideas of Moshe S. Livšic and Some of their Further Developments

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Moshe S. Livšic (1917–2007) was one of the founding fathers of the modern theory of nonselfadjoint and nonunitary operators, discovering both the fundamental notion of the characteristic function and its deep relation to scattering theory and system theory. In the early 1970s M. S. Livšic started considering the problem of linking system theory and the theory of nonselfadjoint operators with Riemannian geometry [5]. This led naturally to developing a spectral theory for tuples of commuting nonselfadjoint operators. The first major breakthrough here was achieved during the three years (1975–1978) that M. S. Livšic spent in Tbilisi and was published in a note in 1978 in the Proceedings of the Georgian Academy of Sciences [2]: M. S. Livšic proved that a pair of commuting nonselfadjoint operators with finite nonhermitian ranks satisfy an algebraic equation with constant coefficients. This developed later into the notion of a commutative operator vessel (an algebraic structure encoding the commutation relations between several nonselfadjoint operators) and its discriminant curve, and an extensive theory of commuting nonselfadjoint operators and related overdetermined multidimensional systems based on connections with algebraic geometry [3]. In algebraic geometry itself it led to a new detailed study of determinantal representations and related topics yielding a proof of the 1958 conjecture of Lax on homogeneous hyperbolic polynomials in three variables [1].

IWOTA 2015 in Tbilisi is an appropriate occasion to survey the genesis of these remarkable ideas and to describe some of their further developments, including the recent generalization of the notion of the vessel to the setting of nonselfadjoint representations of a real Lie algebra [4].

References

[1] J. W. Helton and V. Vinnikov, Linear matrix inequality representation of sets. *Comm. Pure Appl. Math.* **60** (2007), No. 5, 654–674.

[2] M. S. Livšic, Commuting nonselfadjoint operators and solutions of systems of partial differential equations generated by them. (Russian) *Soobshch. Akad. Nauk Gruzin. SSR* **91** (1978), No. 2, 281–284.

[3] M. S. Livšic, N. Kravitsky, A. S. Markus, and V. Vinnikov, Theory of commuting nonselfadjoint operators. Mathematics and its Applications, 332. *Kluwer Academic Publishers Group, Dordrecht*, 1995.

[4] E. Shamovich and V. Vinnikov, Lie Algebra operator vessels. `http://arxiv.org/abs/1209.4224`

[5] L. L. Vaksman and M. S. Livšic, Open geometry, and operator colligations. (Russian) *Ukrain. Geometr. Sb.* No. 15 (1974), 16–35.

Abstracts of Participants’ Talks

Generalized 3-Circular Projections on Function Spaces

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Let X be a complex Banach space and $\mathbb{T}$ denotes the unit circle in the complex plane. A Projection P_0 on X is called a generalized 3-circular projection if there exist distinct $\lambda_1, \lambda_2 \in \mathbb{T}\backslash\{1\}$ of finite order and projections P_1, P_2 on X such that

1. $P_0 \oplus P_1 \oplus P_2 = I$,
2. $P_0 + \lambda_1 P_1 + \lambda_2 P_2$ is a surjective isometry on X.

In this talk, we will give complete description of generalized 3-circular projections on $C(\Omega)$, where Ω is a compact connected Hausdorff space.

Isomorphisms of Operator Systems

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Very recently, Davidson and Kennedy confirmed Arveson's intuition about the fact that boundary representations were the right tool to obtain the C*-envelope of an operator system. Motivated in part by their result, together with joint previous work with D. Farenick, I consider the problem of establishing complete order isomorphism classes of operator systems. The problem is surprisingly hard, even in the case of 3-dimensional operator systems inside a finite-dimensional C*-algebra.

Investigation of Resolvent of Operator-Differential Equations on Semi-Axis

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Let H be a seperable Hilbert space. In the space $H_1 = L_2[H : [0, +\infty]]$ consider a differential operator L generalized by the expression

$$l(y) = (-1)^n (P(x)y^{(n)})^{(n)} + \sum_{j=2}^{2n} Q_j(x) y^{(2n-j)}, \tag{1}$$

With the boundary conditions

$$y^{(l_1)}(0) = y^{(l_2)}(0) = \cdots = y^{(l_n)}(0) = 0, \quad (0 \leqslant l_1 < l_2 < \cdots < l_n \leqslant 2n-1). \tag{2}$$

The main result of this paper is the following.

Theorem. *Under some conditions on the operator coefficients $Q_j(x)$, $j = 2, \ldots, 2n-2$, for sufficiently large $\mu > 0$ exists an inverse operator $R\mu = (L + \mu E)^{-1}$,which is a integral operator with the operator kernel $G(x, \eta; \mu)$,that will be called the Green function of the operator L. $G(x, \eta; \mu)$ is an operator function in H that depends on two variables, x, η $(0 \leqslant x, \eta < \infty)$, the parameter μ, and satisfies the conditions:*

(a) $\dfrac{\partial^k G(x, \eta; \mu)}{\partial \eta k}$, $k = \overline{0, 2n-2}$ *is a strongly continuous operator valued function with respect to variables x, η;*

(b) $\dfrac{\partial^{2n-1} G(x, x+0; \mu)}{\partial \eta^{2n-1}} - \dfrac{\partial^{2n-1} G(x, x-0; \mu)}{\partial \eta^{2n-1}} = (-1)^n P^{-1}(x)$*;*

(c) $(-1)^n (G_\eta^{(n)} P(\eta))_\eta^n + \sum_{j=2}^{2n} G_\eta^{(2n-j)} Q_{j(\eta)} + \mu G = 0$, $\left.\dfrac{\partial^{l_1} G}{\partial \eta^{l_1}}\right|_{\eta=0} = \left.\dfrac{\partial^{l_2} G}{\partial \eta^{l_2}}\right|_{\eta=0} = \cdots = \left.\dfrac{\partial^{l_n} G}{\partial \eta^{l_n}}\right|_{\eta=o} = 0$*;*

(c) $G^*(x, \eta; \mu) = G(\eta; x; \mu)$*;*

(d) $\displaystyle\int_0^\infty \left\| G(x, \eta; \mu) \right\|_H^2 d\eta < \infty.$

Some Operators Inequalities Via Kontorovich Constant

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The Callebaut inequality says that

$$\Big(\sum_{j=1}^{n} x_j^{\frac{1}{2}} y_j^{\frac{1}{2}}\Big)^2 \leqslant \sum_{j=1}^{n} x_j^{\frac{1+s}{2}} y_j^{\frac{1-s}{2}} \sum_{j=1}^{n} x_j^{\frac{1-s}{2}} y_j^{\frac{1+s}{2}} \leqslant \Big(\sum_{j=1}^{n} x_j\Big)\Big(\sum_{j=1}^{n} y_j\Big),$$

where x_j, y_j $(1 \leqslant j \leqslant n)$ are positive real numbers and $s \in [0,1]$ that this is a refinement of the Cauchy–Schwarz inequality. In this talk, we employ some operator techniques to establish some refinements and reverses of the noncommutative Callebaut inequality involving the geometric mean and Hadamard product under some mild conditions.

References

[1] M. Bakherad and M. S. Moslehian, Complementary and refined inequalities of Callebaut inequality for operators. *Linear Multilinear Algebra* **63** (2015), No. 8, 1678–1692; doi: 10.1080/03081087.2014.967234.

[2] M. S. Moslehian, J. S. Matharu, and J. S. Aujla, Non-commutative Callebaut inequality. *Linear Algebra Appl.* **436** (2012), No. 9, 3347–3353.

[3] J. C. Bourin, E. Y. Lee, M. Fujii, and Y. Seo, A matrix reverse Hlder inequality. *Linear Algebra Appl.* **431** (2009), No. 11, 2154–2159.

[4] H. Zuo, G. Shi, and M. Fujii, Refined Young inequality with Kantorovich constant. *J. Math. Inequal.* **5** (2011), No. 4, 551–556.

Function-Theoretic Operator Theory: a Quantized Setting

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A popular framework for the study of nonselfadjoint/nonunitary/nonnormal Hilbert-space operators (where there is no spectral theorem) over the years has been via looking at concrete operators acting on Hilbert spaces of analytic functions. Perhaps the first instance of this approach was the study of the shift operator $f(z) \mapsto zf(z)$ on the Hardy spec over the unit disk in the complex plane: Beurling's Theorem characterizes quite explicitly the lattice of invariant subspaces for the shift and compression of the shift to its coin variant subspaces is the starting point for the model theory for general classes of contraction operators associated with the names of Livsiš, de Branges–Rovnyak, and Sz.-Nagy–Foias. A parallel if less complete theory has emerged where one replaces the Hardy space with a weighted Bergman space on the unit disk and the class of contract operators with the more structured class of n-hypercontraction operators. There have been multivariable extensions of this theme as well, whereby the disk is replaced by a domain in $\mathbb{C}^d$ (e.g., the ball or the polydisk), and one studies commutative operator tuples (modeled as multiplication by the coordinate functions on a multivariable weighted hardy or Bergman space over such a domain). This talk will discuss recent developments leading to a quantized version of this framework, whereby the ambient functional-model Hilbert space consists of functions of matrix or operator rather than scalar variables, and the associated operator theory concerns models for various classes of possibly noncommutative operator tuples acting on a Hilbert space. Such a program can already be seen in the model theory for row contractions as well as in some of the more recent work of Gelu Popescu. Various aspects of the talk are drawn from joint work of the speaker with Vladimir Bolotnikov (College of William & Mary), Gregory Marx (Virginia Tech), and Victor Vinnikov (Ben Gurion University).

Lower Bounds for Regularised Determinants and an Application to Estimating Spectral Variation

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Let p be a positive integer. Given an operator A belonging to the Schatten class S_p, consider the corresponding regularized determinant of order p, that is,

$$\det_p(1-A) = \prod_{n=1}^{\infty}(1-\lambda_n)\exp\Big(\sum_{k=1}^{p-1}\frac{\lambda_n^k}{k}\Big),$$

where $(\lambda_n)_{n\in\mathbb{N}}$ denotes the sequence of eigenvalues of A. The regularized determinant plays an important role in perturbation theory, as $z \longmapsto \det_p(1-zA)$ is an entire function of finite order with its zeros coinciding with the reciprocals of the eigenvalues of A.

In this talk I will report on new lower bounds of the form

$$\left|\det_p(1 - z^{-1}A)\right| \geqslant \exp\Big(- c_p \frac{\|A\|_p^p}{\operatorname{dist}(z, \sigma(A))^p}\Big) \quad (\forall\, z \in \mathbb{C}\backslash\sigma(A))$$

where $\|A\|_p$ denotes the Schatten norm of A and $\operatorname{dist}(z, \sigma(A))$ denotes the distance of z to the spectrum $\sigma(A)$ of A. The constant c_p does not depend on A and can be chosen to be

$$c_p = \begin{cases} 1 & \text{if } p = 1, \\ \dfrac{p-1}{p} & \text{if } p > 1. \end{cases}$$

I will also explain how this lower bound can be used to obtain explicit upper bounds for the operator norm $\|(zI - A)^{-1}\|$ of the resolvent of a Schatten class operator A expressible in terms of the Schatten norm and the distance of z to $\sigma(A)$ only. These bounds in turn yield explicit upper bounds for the spectral variation of two Schatten class operators, that is, the Hausdorff distance of their spectra, in terms of their Schatten norms and the operator norm of their difference.

The resulting resolvent and spectral variation bounds improve existing bounds of this form obtained by different means (see [1–3]). Moreover, unlike the methods used in [1–3], the approach described above easily generalizes to the Banach space case.

References

[1] O. F. Bandtlow, Estimates for norms of resolvents and an application to the perturbation of spectra. *Math. Nachr.* **267** (2004), 3–11.

[2] M. I. Gil', Operator functions and localization of spectra. Lecture Notes in Mathematics, 1830. *Springer-Verlag, Berlin*, 2003.

[3] A. Pokrzywa, On continuity of spectra in norm ideals. *Linear Algebra Appl.* **69** (1985), 121–130.

Generic Rank-k Perturbations of J-Hamiltonian Matrices

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This talk deals with the effect of generic but structured low rank perturbations on the Jordan structure and sign characteristic of matrices that have structure in an indefinite inner product space. In particular we shall be concerned with J-Hamiltonian matrices. This is a follow-up of earlier results in which the effect of rank one perturbations was considered.

References

[1] L. Batzke, Chr. Mehl, A.C.M. Ran and L. Rodman: Generic rank-K perturbations of structured matrices. (to appear).

[2] Chr.Mehl, V. Mehrmann, A. C. M. Ran, and L. Rodman, Eigenvalue perturbation theory of classes of structured matrices under generic structured rank one perturbations. *Linear Algebra Appl.* **435** (2011), No. 3, 687–716.

A Functional Model of a Symmetric Semi-Bounded Operator in Inverse Problems on Manifolds

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We deal with the problem of reconstruction of a Riemannian manifold Ω from its boundary (dynamical and/or spectral) inverse data. It is shown that the problem can be solved by constructing a functional model of a symmetric semi-bounded operator L_0 determined by the inverse data. A basic element of the construction is the so-called *wave spectrum* Ω_{L_0} of L_0, which is introduced via the trajectories of a dynamical system governed by the wave equation $u_{tt} + L_0^* u = 0$. The spectrum Ω_{L_0} is endowed with relevant topology and metric, which turn it to a Riemannian manifold $\tilde{\Omega}$ such that $\tilde{\Omega} \overset{\text{isom}}{=} \Omega$ holds. Thus, $\tilde{\Omega}$ provides the solution to the reconstruction problem.

References

[1] M. I. Belishev, Recent progress in the boundary control method. *Inverse Problems* **23** (2007), No. 5, R1–R67.

[2] M. I. Belishev, A unitary invariant of a semi-bounded operator in reconstruction of manifolds. *J. Operator Theory* **69** (2013), No. 2, 299–326.

[3] M. I. Belishev and M. N. Demchenko, Elements of noncommutative geometry in inverse problems on manifolds. *J. Geom. Phys.* **78** (2014), 29–47.

Generalized Donoghue Classes and Conservative L-Systems

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The focus of our study are the connections among: (i) the Livšic class of functions $s(z)$ that are the characteristic functions of a densely defined symmetric operators $\dot{A}$ with deficiency indices $(1,1)$; (ii) the characteristic functions $S(z)$ (the Möbius transform of $s(z)$) of a maximal dissipative extension T of $\dot{A}$ determined by the von Neumann parameter κ; (iii) the transfer functions $W_\Theta(z)$ of conservative L-systems Θ with the main operator T of the system. It is shown that under some natural hypothesis $S(z)$ and $W_\Theta(z)$ are reciprocal to each other. We establish that the impedance function of a conservative L-system coincides with the function from the Donoghue class if and only if the von Neumann parameter vanishes ($\kappa = 0$). Moreover, we introduce the generalized Donoghue class and provide the criteria for an impedance function to belong to this class. We also obtain the representation of a function from this class via the Weyl–Titchmarsh function.

The talk is based on joint work with K. A. Makarov and E. Tsekanovskiĭ (see also references below).

References

[1] S. Belyi, K. A. Makarov, and E. Tsekanovskiĭ, Conservative L-systems and the Livšic function. *Methods Funct. Anal. Topol.*, 2015 (to appear).

[2] K. A. Makarov and E. Tsekanovskiĭ, On the Weyl-Titchmarsh and Livic functions. *Spectral analysis, differential equations and mathematical physics: a festschrift in honor of Fritz Gesztesy's 60th birthday*, 291–313, Proc. Sympos. Pure Math., 87, *Amer. Math. Soc.*, Providence, RI, 2013.

[3] Yu. Arlinskiĭ, S. Belyi, and E. Tsekanovskiĭ, Conservative realizations of Herglotz-Nevanlinna functions. Operator Theory: Advances and Applications, 217. *Birkhäuser/ Springer Basel AG, Basel*, 2011.

Operator Positivity and Analytic Models of Commuting Tuples of Operators

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We study analytic models of operators of class $C_{\cdot 0}$ with natural positivity assumptions. In particular, we prove that for an m-hypercontraction $T \in C_{\cdot 0}$ on a Hilbert space $\mathscr{H}$, there exists a Hilbert space $\mathscr{E}$ and a partially isometric multiplier $\theta \in \mathscr{M}(H^2(\mathscr{E}), A^2_m(\mathscr{H}))$ such that

$$\mathscr{H} \cong \mathscr{Q}_\theta = A^2_m(\mathscr{H}) \ominus \theta H^2(\mathscr{E}) \text{ and } T \cong P_{\mathscr{Q}_\theta} M_z\big|_{\mathscr{Q}_\theta},$$

where A^2_m is the weighted Bergman space and H^2 is the Hardy space over the unit disc $\mathbb{D}$. We then proceed to study and develop analytic models for doubly commuting n-tuples of operators and investigate their applications to joint shift co-invariant subspaces of reproducing kernel Hilbert spaces over polydisc. In particular, we completely analyze doubly commuting quotient modules of a large class of reproducing kernel Hilbert modules, in the sense of Arazy and English, over the unit polydisc $\mathbb{D}^n$.

References

[1] C.-G. Ambrozie, M. Engliš, and V. Müller, Operator tuples and analytic models over general domains in $\mathbb{C}^n$. *J. Operator Theory* **47** (2002), No. 2, 287–302.

[2] T. Bhattacharyya, E. K. Narayanan, and J. Sarkar, Analytic model of doubly commuting contractions. `http://arxiv.org/abs/1310.0950`.

Efficient Cyclic Reduction for QBDs with Rank Structured Blocks

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The cyclic reduction is classical algorithm originally developed by G. H. Golub and R. W. Hockney for the solution of some structured linear systems. More recently this method has been generalized in order to find the solution of a particular class of matrix equations defined by power series [1].

One of the main applications of this iteration is the solution of QBD Markov Chains, where a matrix equation of the form $A + BX + CX^2 = 0$ arises. We consider the case where A, B and C are rank structured, and in particular the case where every off-diagonal submatrix has the rank bounded by a small integer (the so called quasiseparability rank). This is often encountered in applications where these matrices may be tridiagonal, banded or, more generally, quasiseparable [2].

The cyclic reduction defines an iteration on three matrices A_k, B_k, C_k where $A_0 = A$, $B_0 = B$ and $C_0 = C$. The matrices are obtained at each step from the previous ones through the computation of matrix products and the solution of linear systems. We show that even if the quasiseparability property is not preserved during the iteration, the matrices A_k, B_k and C_k are such that every off-diagonal submatrix has singular values with exponential decay dependent only on the spectral property of a certain matrix function.

We exploit this fact to find accurate approximations of A_k, B_k and C_k in the set of quasiseparable matrices in order to speed up the computation. We discuss the implementation of this strategy and we show that we are able to reach very accurate results in $O(n \log n^2)$ flops per step instead of the classical $O(n^3)$ flops. Moreover, our approach allows for the analysis of QBDs with a large number of states per blocks where the memory usage is a concern if the storage of full $n \times n$ matrices is needed. We show that our approach only needs $O(n \log n)$ memory.

References

[1] D.A. Bini and B. Meini, The cyclic reduction algorithm: from Poisson equation to stochastic processes and beyond. In memoriam of Gene H. Golub. *Numer. Algorithms* **51** (2009), No. 1, 23–60.

[2] M. Miyazawa, Tail decay rates in double QBD processes and related reflected random walks. *Math. Oper. Res.* **34** (2009), No. 3, 547–575.

Laplace–Beltrami Equation on Hypersurfaces and Γ-Convergence

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Let us consider heat conduction by an "isotropic" media, governed by the Laplace equation with the classical Dirichlet–Neumann mixed boundary conditions on the boundary in the layer domain $\Omega^\varepsilon := \mathscr{C} \times (-\varepsilon, \varepsilon)$ of a thickness 2ε. More precisely we impose zero Dirichlet and non-zero Neumann data on the corresponding parts of the boundary

$$\begin{aligned}\Delta_{\Omega^\varepsilon} T(x,t) &= f(x,t), \ \ (x,t) \in \mathscr{C} \times (-\varepsilon, \varepsilon),\\ T^+(x,t) &= 0, \ \ (x,t) \in \partial\mathscr{C} \times (-\varepsilon, \varepsilon),\\ \pm(\partial_t T)^+(x, \pm\varepsilon) &= q(x, \pm\varepsilon), \ \ x \in \mathscr{C},\end{aligned}$$

where $\pm\partial_t = \partial_v$ represents the normal derivative on the surfaces $\mathscr{C} \times \{\pm\varepsilon\}$. Here $\mathscr{C} \subset \mathscr{S}$ is a smooth subsurface of a closed hypersurface $\mathscr{S}$ with smooth nonempty boundary $\partial\mathscr{C}$.

The suggested approach is based on the fact that the Laplace operator $\Delta_{\Omega^\varepsilon} = \partial_1^2 + \partial_2^2 + \partial_3^2$ is represented as the sum of the Laplace–Beltrami operator on the mid-surface and the square of the transversal derivative:

$$\Delta_{\Omega^\varepsilon} T = \sum_{j=1}^{4} \mathscr{D}_j^2 T = \Delta_{\mathscr{C}} T + \partial_t^2 T.$$

In the report we will review what happens with the above mentioned mixed boundary value problem when the thickness of the layer converges to zero in the sense of Γ-convergence. It is proved that the limit coincides with the Dirichlet BVP for the Laplace–Beltrami equation, which is described explicitly. It is shown how the Neumann boundary conditions from the initial BVP transform during the Γ-limit and wanders to the right hand side of the limit BVP. For this we apply the variational formulation and the calculus of Günter's tangential differential operators on a hypersurface and layers, which allow global representation of basic differential operators and of corresponding boundary value problems in terms of the standard Euclidean coordinates of the ambient space $\mathbb{R}^n$.

A similar results on Γ-limits of BVP for the Laplace equation, but with different approach and for different boundary conditions, was obtained in the papers [1], [2].

References

[1] G. Friesecke, R. D. James, and S. Müller, A theorem on geometric rigidity and the derivation of nonlinear plate theory from three-dimensional elasticity. C*omm. Pure Appl. Math.* **55** (2002), No. 11, 1461–1506.

[2] G. Friesecke, R.D. James, M. G. Mora, and S. Müller, Derivation of nonlinear bending theory for shells from three-dimensional nonlinear elasticity by Gamma-convergence. *C. R. Math. Acad. Sci. Paris* **336** (2003), No. 8, 697–702.

Subnormality of Composition Operators: Exotic Examples. I

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In my talk I will address the problem of subnormality of unbounded composition operators (in L^2-spaces) over one-circuit directed graphs. I will show how recently developed techniques of studying the subnormality of unbounded composition operators (cf. [1]) can be applied in this particular context. The topic is motivated by the question of existence of a non-hyponormal composition operator over locally finite directed graph which generates Stieltjes moment sequences (in case of non-locally finite graphs, a solution was given in [2]) and is related to highly non-trivial problems concerning N-extremal measures, indeterminate moment problems, etc.

This is the first part of a two-part talk. The other part will be presented by Z. J. Jabłoński in the same Thematic Session.

The talk is based on joint work with Z. J. Jabłoński, I. B. Jung, and J. Stochel (cf. [3]).

References

[1] P. Budzyński, Z. J. Jabłoński, I. B. Jung, and J. Stochel, Unbounded subnormal composition operators in L^2-spaces. *J. Funct. Anal.*, 2015; doi:10.1016/j.jfa.2015.01.024.

[2] Z. J. Jabłoński, I. B. Jung, and J. Stochel, A non-hyponormal operator generating Stieltjes moment sequences. *J. Funct. Anal.* **262** (2012), No. 9, 3946–3980.

[3] P. Budzyński, Z. J. Jabłoński, I. B. Jung, J. Stochel, Subnormality of unbounded composition operators over one-circuit directed graphs: exotic examples. *preprint*, 2015.

Composition Operators in L^2-Spaces Via Inductive Limits

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One way of study unbounded operators are inductive limits of operators (cf. [4, 5]). In particular, they can be useful when dealing with unbounded composition operators in L^2-spaces. One of the aims of this talk is to discuss the questions of dense definiteness and boundedness of this class of operators. These two properties have characterization (cf. [1, 3, 7]) which in a more concrete situations seem difficult to apply. For example, this is the case of a composition operator induced by an infinite matrix in $L^2(\mu_G)$, where μ_G is the gaussian measure on $\mathbb{R}^\infty$. One possible approach to deal with these problems is a technique based on inductive limits. Tractable criteria for the above mentioned properties will be presented. This is possible if the L^2-space (in which a given composition operator acts) is an inductive limit of L^2-spaces with underlying measure spaces forming a projective system. In this case both the dense definiteness and boundedness can be described in terms of asymptotic behaviour of appropriate Radon–Nikodym derivatives.

The inductive limit technique can also be applied when discussing the question of subnormality and cosubnormality of composition operators induced by linear transformations of $\mathbb{R}^k$ (cf. [2, 6, 8]).

References

[1] P. Budzyński, Z. J. Jabłoński, I. B. Jung, and J. Stochel, A multiplicative property characterizes quasinormal composition operators in L^2-spaces. *J. Math. Anal. Appl.* **409** (2014), No. 1, 576–581.

[2] P. Budzyński, Z. J. Jabłoński, I. B. Jung, and J. Stochel, Unbounded subnormal composition operators in L^2-spaces. *J. Funct. Anal.* (to appear).

[3] J. T. Campbell and W. E. Hornor, Seminormal composition operators. *J. Operator Theory* **29** (1993), No. 2, 323–343.

[4] J. Janas, Inductive limit of operators and its applications. *Studia Math.* **90** (1988), No. 2, 87–102.

[5] A. V. Marchenko, Selfadjoint differential operators with an infinite number of independent variables. (Russian) *Mat. Sb. (N.S.)* **96(138)** (1975), 276–293, 343–344.

[6] W. Mlak, Operators induced by transformations of Gaussian variables. *Ann. Polon. Math.* **46** (1985), 197–212.

[7] E. A. Nordgren, Composition operators on Hilbert spaces. *Hilbert space operators (Proc. Conf., Calif. State Univ., Long Beach, Calif., 1977)*, pp. 37–63, Lecture Notes in Math., 693, *Springer, Berlin*, 1978.

[8] J. Stochel, Seminormal composition operators on L^2 spaces induced by matrices. *Hokkaido Math. J.* **19** (1990), No. 2, 307–324.

Conformal Spectral Stability Estimates for the Dirichlet Laplacian

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This talk is devoted to stability estimates for the eigenvalues of the Dirichlet Laplacian

$$-\Delta f = -\Big(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\Big), \quad (x,y) \in \Omega, \quad f|_{\partial\Omega} = 0.$$

It is known that in a bounded plane domain $\Omega \subset \mathbb{C}$ the spectrum of the Dirichlet Laplacian is discrete and can be written in the form of a non-decreasing sequence

$$0 < \lambda_1[\Omega] \leqslant \lambda_2[\Omega] \leqslant \cdots \leqslant \lambda_n[\Omega] \leqslant \cdots,$$

where each eigenvalue is repeated as many times as its multiplicity.

Let, for $\tau > 0$, F_τ be the set of all mappings φ of the unit disc $\mathbb{D}$ of the Sobolev class $L^{1,\infty}(\mathbb{D})$ such that $\|\nabla\varphi\|_{L^\infty(\mathbb{D})} \leqslant \tau$, $\operatorname*{ess\,inf}_{\mathbb{D}} |\det \nabla\varphi| \geqslant \frac{1}{\tau}$.

Theorem ([2]). *For any $\tau > 0$ there exists $A_\tau > 0$ such that for any $\varphi_1, \varphi_2 \in F_\tau$ and for any $n \in \mathbb{N}$*

$$|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]| \leqslant c_n A_\tau \|\varphi_1 - \varphi_2 \mid L^{1,\infty}(\mathbb{D})\|,$$

where $\Omega_1 = \varphi_1(\mathbb{D})$, $\Omega_2 = \varphi_2(\mathbb{D})$ and $c_n = \max\{\lambda_n^2[\Omega_1], \lambda_n^2[\Omega_2]\}$.

We consider bounded simply connected plane domains $\Omega \subset \mathbb{C}$ for which there exist conformal mappings $\varphi : \mathbb{D} \to \Omega$ of the Sobolev class $L^{1,p}(\mathbb{D})$ for some $p > 2$. For such domains we improve the above estimate.

Recall that for any $2 \leqslant q < \infty$ the Sobolev inequality

$$\|f \mid L^q(\mathbb{D})\| \leqslant C(q)\|\nabla f \mid L^2(\mathbb{D})\|$$

holds for any function $f \in W_0^{1,2}(\mathbb{D})$.

Theorem. Let $\varphi_1 : \mathbb{D} \to \Omega_1$, $\varphi_2 : \mathbb{D} \to \Omega_2$ be conformal mappings. Suppose that $|\varphi_1'|, |\varphi_2'| \in L^p(\mathbb{D})$ for some $2 < p \leqslant \infty$.

Then for any $n \in \mathbb{N}$

$$|\lambda_n[\Omega_1] - \lambda_n[\Omega_2]| \leqslant c_n \Big[C\Big(\frac{4p}{p-2}\Big)\Big]^2 \Big(\|\varphi_1' \mid L^p(\mathbb{D})\| + \|\varphi_2' \mid L^p(\mathbb{D})\| \Big) \|\varphi_1 - \varphi_2 \mid L^{1,2}(\mathbb{D})\|,$$

where $\Omega_1 = \varphi_1(\mathbb{D})$, $\Omega_2 = \varphi_2(\mathbb{D})$.

Joint work with V. Goldshtein and A. Ukhlov.

Detailed exposition of the results is contained in [1].

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References

[1] V. I. Burenkov, V. Goldshtein, A. Ukhlov, Conformal spectral stability estimates for the Dirichlet. *Math. Nachr.*, 2015, (accepted).

[2] P. D. Lamberti and M. Lanza de Cristoforis, A global Lipschitz continuity result for a domain dependent Dirichlet eigenvalue problem for the Laplace operator. *Z. Anal. Anwendungen* **24** (2005), No. 2, 277–304

On General Domain Truncated Correlation and Convolution Operators with Finite Rank

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Truncated correlation and convolution operators is a general operator-class containing popular operators such as Toeplitz (Wiener–Hopf), Hankel and finite interval convolution operators as well as small and big Hankel operators in several variables. We completely characterize the symbols for which such operators have finite rank, and develop methods for determining the rank in concrete cases. Such results are well known for the one-dimensional objects, the first discovered by L. Kronecker during the 19th century. We show that the results for the multidimensional case differ in various key aspects.

Operator-Lipschitz Estimates for the Singular Value Functional Calculus

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We consider a functional calculus for compact operators, acting on the singular values rather than the spectrum, which appears frequently in applied mathematics. Necessary and sufficient conditions for this singular value functional calculus to be Lipschitz-continuous with respect to the Hilbert-Schmidt norm are given. We also provide sharp constants.

Diffraction by a Half-Plane with Different Face Impedances on an Obstacle Perpendicular to the Boundary

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We study two wave diffraction problems, modeled by the Helmholtz equation, in a half-plane with a crack characterized by impedance boundary conditions. Conditions on the wave number and impedance parameters are found to ensure the well-posedness of the problems in Sobolev spaces. This is done by an appropriate combination of general operator theory, Fredholm theory, potential theory and boundary integral equation methods. This combination of methods leads also to integral representations of solutions. Moreover, in Sobolev spaces, a range of smoothness parameters is obtained in which the solutions of the problems are valid.

The talk is based on joint work with the co-author David Kapanadze (from Andrea Razmadze Mathematical Institute, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia).

Uniqueness of Solutions of the Hypoelliptic Differential Equations

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Radiation conditions were first derived by A. Sommerfeld for Helmholtz operator [1] and subsequently were generalized in the following papers [2, 3]. Here are obtained Sommerfeld type conditions at infinity for polymetaharmonic equations, which ensure uniqueness of solutions in $\mathbb{R}^n$. In the paper [2] is studied uniqueness of solution of the polymetaharmonic equation, where characteristic polynomial has multiple zeros.

In the monograph [4] were obtained radiation conditions for hypoelliptic differential equations, where characteristic polynomials have real simple zeros.

We generalize the results obtained in [4] and consider the case when the corresponding characteristic polynomials of the hypoelliptic differential equations have real multiple zeros. We investigate asymptotic properties at infinity of fundamental solutions of the hypoelliptic differential equations. On the basis of asymptotic analysis of fundamental solution we find conditions at infinity, which ensure that these equations are uniquely solvable.

References

[1] A. Sommerfeld, Partial Differential Equations in Physics. Translated by Ernst G. Straus. *Academic Press, Inc., New York, N. Y.*, 1949.

[2] B. Paneah, On existence and uniqueness of the solution of n-metaharmonic equation in unbounded space. *Mosc. State Univ. Vestnic, Math. Mech.*, No. 5, 1959, 123–135.

[3] I. Vekua, On metaharmonic functions. *Trudy Tbiliss. Mat. Inst. Razmadze* **12** (1943), 105–174.

[4] B. R. Vaĭnberg, Asymptotic methods in equations of mathematical physics. Translated from the Russian by E. Primrose. *Gordon & Breach Science Publishers, New York*, 1989.

Localized Boundary-Domain Integral Equations Approach for Problems of the Theory of Electro-Magneto-Elasticity for Inhomogeneous Solids

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We consider the three-dimensional Dirichlet and Robin boundary-value problems (BVPs) of electro-magneto-elasticity for anisotropic inhomogeneous solids and using the localized parametrix approach develop the generalized potential method. Using Green's integral representation formula and properties of the localized layer and volume potentials we reduce the Dirichlet and Robin BVPs to the corresponding localized boundary-domain integral equations (LBDIE) systems.

First we establish the equivalence between the original boundary value problems and the corresponding LBDIE systems. Afterwards, we establish that the localized boundary-domain integral operators obtained belong to the Boutet de Monvel algebra and with the help of the Vishik–Eskin theory, based on the factorization method (Wiener–Hopf method), we investigate corresponding Fredholm properties and prove invertibility of the localized operators in appropriate function spaces.

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Generalization of Toeplitz Operators on Bergman and Hardy Spaces on Bounded Domains

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The talk is about algebraic properties of Laurent and Toeplitz operators on Bergman and Hardy spaces associated to general bounded domains in the complex plane. Many results of classification of two operators have been proved mainly so far only for the case of the unit disc. Simply connected bounded domains are conformally equivalent to the unit disc via the Riemann mapping function and similarly finitely connected bounded domains are proper holomorphically mapped onto the unit disc via the Ahlfors map. So by studying the boundary behavior of biholomorphic and proper holomorphic mappings, we generalize these results to general bounded domains. First we construct an orthonormal basis for $L^2(b\Omega)$ space of a bounded domain Ω in terms of the classical kernel functions in potential theory and do the almost same procedure as in [1]. In particular, we classify Laurent operators and Toeplitz operators on Hardy spaces in terms of the kernel functions and the Ahlfors maps and then in terms of Laurent matrices and Toepliz matrices. Secondly, by using the transformation formulas for the Toeplitz operators under the Riemann map and the Ahlfors map, we find algebraic properties of Toeplitz operators on both Bergman and Hardy spaces associated to general bounded domains. This is a new development to study on Laurent and Toeplitz operators on bounded domains.

References

[1] A. Brown and P. R. Halmos, Algebraic properties of Toeplitz operators. *J. Reine Angew. Math.* **213** (1963/1964), 89–102.

Multivariable Aluthge Transforms

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We introduce two natural notions of multivariable Aluthge transforms (Cartesian and polar) for 2-variable weighted shifts and study their basic properties. We first prove that neither the

Cartesian nor the polar Aluthge transform preserves (joint) hyponormality, in sharp contrast with the 1-variable case. Second, we investigate the existence of necessary and sufficient conditions for the invariance of (joint) hyponormality for commuting 2-variable weighted shifts under these transforms. Third, we study how the the Taylor and Taylor essential spectra of 2-variable weighted shifts behave under the Cartesian and polar Aluthge transforms. Finally, we discuss the class of spherically quasinormal 2-variable weighted shifts, which are the fixed points for the polar Aluthge transform.

The talk is based on joint work with Jasang Yoon.

The Division Algorithm in Truncated Moment Problems

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For a degree $2n$ complex sequence $\gamma \equiv \gamma^{(2n)} = \{\gamma_{ij}\}_{i,j\in Z_+, i+j\leqslant 2n}$ to have a representing measure μ, it is necessary for the associated moment matrix $M(n)$ to be positive semidefinite, and for the algebraic variety associated to γ, $\mathscr{V}_\gamma \equiv \mathscr{V}(M(n))$, to satisfy $\operatorname{rank} M(n) \leqslant \operatorname{card} \mathscr{V}_\gamma$ as well as the following *consistency* condition: if a polynomial $p(z,\overline{z}) \equiv \sum_{ij} a_{ij}\overline{z}^i z^j$ of degree at most $2n$ vanishes on $\mathscr{V}_\gamma$, then the *Riesz functional* $\Lambda(p) \equiv p(\gamma) := \sum_{ij} a_{ij}\gamma_{ij} = 0$.

Positive semidefiniteness, recursiveness, and the variety condition of a moment matrix are necessary and sufficient conditions to solve the quadratic ($n = 1$) and quartic ($n = 2$) moment problems. Also, positive semidefiniteness, combined with consistency, is a sufficient condition in the case of *extremal* moment problems, i.e., when the rank of the moment matrix (denoted by r) and the cardinality of the associated algebraic variety (denoted by v) are equal.

For extremal sextic moment problems, verifying consistency amounts to having good representation theorems for sextic polynomials in two variables vanishing on the algebraic variety of the moment sequence. We obtain such representation theorems using the Division Algorithm from algebraic geometry. As a consequence, we are able to complete the analysis of extremal sextic moment problems.

The talk is based on joint work with Seonguk Yoo.

On One Quasi-Stationary Nonlinear Mathematical Model for Leak Localization in the Branched Gas Pipeline

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Nowadays pipelines become the main practical means for natural gas transportation worldwide but it should be noted that at the same time the gas delivery infrastructure is rapidly aging. The main fault of the outdated pipelines is leak and as a consequence deterioration of environment. In pipeline networks that transport gas or oil leaks may occur at any time and location, therefore, timely leak detection and localization is important for the safe operation of pipelines their impacts on environment. So elaboration the leak detection and location system is an urgent and sensitive issue of nowadays. There are many different approaches and techniques for leak detection and location but from existing methods the mathematical modelling with hydrodynamic method is more acceptable as it is very cheap and reliable method with high sensitive and operative features. Recently, many gas flow mathematical models have been developed and a number are using by the gas-liquid industry but as accounting practices have shown none of them are universal. For this reason investigation of the mathematical models adequate describing the real non-stationary not isothermal processes processing in the branched pipeline systems by analytical methods are actual. In the present paper pressure and gas flow rate distribution in the branched pipeline on the bases of one quasi-stationary nonlinear mathematical model is investigated. For realization of this purpose the system of partial differential equations describing gas quasi-stationary flow in the branched pipeline was studied. We have found effective solutions of this quasi-stationary nonlinear partial differential equations (pressure and gas flow rate distribution in the branched pipeline) for leak detection in horizontal branched pipeline. for learning the affectivity of the method quite general test was created. Preliminary data of numerical calculations have shown efficiency of the suggested method. Some results of numerical calculations defining localization of gas escape for the inclined pipeline are presented. The results of calculations on the basis of observation data have shown that the performed simulations were much closer to the results of observation.

Polynomial Zigzag Matrices, Dual Minimal Bases, and Applications

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Polynomial zigzag matrices are a class of highly structured matrix polynomials that have been introduced very recently in [1]. Zigzag matrices can be thought as a nontrivial extension of the singular blocks of the classical Kronecker canonical form of matrix pencils and are a particular type of polynomial minimal bases of rational vector subspaces. It is well known that polynomial minimal bases play a fundamental role in multivariable linear system theory [2]–[4], although their origin is much older since Plemelj in 1908 and the Georgian mathematicians Muskhelishvili and Vekua in 1943 already realized the importance of this concept and performed substantial developments on minimal bases. The main goal of this talk is to describe in detail the new class of polynomial zigzag matrices and to show how they are used to solve the inverse degree problem of dual minimal bases. In the last part of the talk, we describe briefly some other applications of zigzag matrices in the solution of modern problems in the theory of matrix polynomials.

References

[1] F. De Terán, F. M. Dopico, D. S. Mackey, and P. Van Dooren, Polynomial zigzag matrices, dual minimal bases, and the realization of completely singular polynomials, 2015 *submitted*; Available as MIMS EPrint 2015.7, Manchester Institute for Mathematical Sciences, The University of Manchester.

[2] G. D. Forney, Jr., Minimal bases of rational vector spaces, with applications to multivariable linear systems. *SIAM J. Control* **13** (1975), 493–520.

[3] T. Kailath, Linear systems. Prentice-Hall Information and System Sciences Series. *Prentice-Hall, Inc., Englewood Cliffs, N.J.*, 1980. *Linear Systems*. Prentice Hall, Englewood Cliffs, NJ, 1980.

[4] W. A. Wolovich, Linear multivariable systems. Applied Mathematical Sciences, Vol. 11. *Springer-Verlag, New York-Heidelberg*, 1974.

On the Kernels of Generalized Toeplitz Plus Hankel Operators

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The aim of the present work is to study generalized $T(a) + H_\alpha(b)$ generated by a shift α and functions a, b are considered on the Hardy spaces $H^p(\mathbb{T})$, $1 < p < \infty$. These operators are similar to the classical Toeplitz plus Hankel operators but the flip operator $Jf(t) = (1/t)f(1/t)$ in the definition of Hankel operator is replaced by another operator J_α generated by a linear fractional shift α,

$$\alpha(t) := \frac{t-\beta}{\overline{\beta}t-1}\,, \quad t \in \mathbb{T},$$

where β is a complex number such that $|\beta| > 1$, so that the shift α changes the orientation of the circle $\mathbb{T}$. Areas of particular interest to us are the kernels and cokernels of such operators and we derive an explicit description of these spaces in the case where the generating functions a and b belong to the space L^∞ and satisfy the additional algebraic relation

$$a(t)\,a(\alpha(t)) = b(t)\,b(\alpha(t)), \quad t \in \mathbb{T}. \tag{3}$$

Generalized Toeplitz plus Hankel operators have been previously considered in [1], [2]. The approach of [1], [2] is based on a special factorization of the operators in question, which in turn involves factorizations of matrix functions. In particular, the authors express the kernel and cokernel dimensions of $T(a) + H_\alpha(b)$ in terms of the partial indices of some matrix valued functions. On the other hand, it is well known that for an arbitrary matrix function, the computation of its partial indices is a very demanding task. Moreover, in the most cases this is not possible at all. However, if the generating functions a and b satisfy condition (3), one can describe the kernels of the operators $T(a) + H_\alpha(b)$ without factorization of any matrix function. This approach modeled after [3] where classical Toeplitz plus Hankel operators are considered.

References

[1] V. G. Kravchenko, A. B. Lebre, and J. S. Rodríguez, Factorization of singular integral operators with a Carleman shift via factorization of matrix functions: the anticommutative case. *Math. Nachr.* **280** (2007), No. 9-10, 1157–1175.

[2] V. G. Kravchenko, A. B. Lebre, and J. S. Rodríguez, Factorization of singular integral operators with a Carleman backward shift: the case of bounded measurable coefficients. *J. Anal. Math.* **107** (2009), 1–37.

[3] V. D. Didenko and B. Silbermann, Structure of kernels and cokernels of Toeplitz plus Hankel operators. *Integral Equations Operator Theory* **80** (2014), No. 1, 1–31.

Statistical Approximation Properties of Stancu Type q-Baskakov–Kantorovich Operators

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In the present paper, we consider Stancu type generalization of Baskakov–Kantorovich operators based on the q-integers and obtain statistical and weighted statistical approximation properties of these operators. Rates of statistical convergence by means of the modulus of continuity and the Lipschitz type function are also established for said operators. Finally, we construct a bivariate generalization of the operator and also obtain the statistical approximation properties.

Spectral Sets and Generalized Inverses

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If an operator is not invertible, sometimes it is convenient looking for a closed subspace such that the reduction of the operator to that subspace is invertible. We are interested in algebraic conditions characterizing such situation. These characterizations will give us a sort of generalization of the notion of invertibility. In this talk we follow a spectral approach to generalized inverses considering the subspace determined by the range of the spectral projection associated with an operator and a spectral set containing the point 0. We compare the cases when 0 is a simple pole of the resolvent function, 0 is a pole of finite order of the resolvent function, 0 is an isolated point of the spectrum, and 0 is contained in a circularly isolated spectral set.

On the Inverse Problem for Strong Hyperbolic Pencils

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We consider the following problem. Let $L(\lambda)$ be a quadratic operator pencil

$$L(\lambda) = \lambda^2 I + \lambda B + C, \tag{1}$$

where B is a uniformly positive operator and C is positive operator in a Hilbert space H. Let A be a linearizator of $L(\lambda)$. If $L(\lambda)$ is a strong hyperbolic operator pencil that is

$$(Bf, f)^2 - 4(Cf, f)(f, f) \geqslant \delta(f, f) \quad (\delta > 0),$$

then there exists a real number $\alpha > 0$ such that $L(\lambda)$ is reduce to a pencil of type

$$M(\mu) = \mu^2 I + \mu F - D, \quad F = F^*, \quad D > 0, \quad \mu = \lambda - \alpha. \tag{2}$$

We shown that a linearizator of $M(\mu)$ is a selfadjoint operator in $H_+ \oplus H_-$. It is admit an extension of the class of quadratic operator pencils for which so called "inverse linearization problem" is solved.

The following theorem is our main result in this direction.

Theorem. *Let A be a uniformly positive operator in a Krein space $\widehat{H} = H_+ \oplus H_-$. Then $A + \beta I$ is a linearizator of a pencil of type* (2) *if and only if* $\dim H_+ = \dim H_-$ *and* $\beta > \max\{\mu : \mu \in \sigma(A)\}$.

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Computing Matrix Symmetrizers, Leading to Open Matrix Optimization Problems

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I discuss ways to compute symmetric matrix factorizations of a given matrix $A \in F_{n,n}$.

In 1910, Frobenius showed that every matrix $A_{n,n}$ over any field F is the product of two symmetric ones, i.e., $A = S_1 \cdot S_2$, where one of the S_i can be chosen nonsingular.

To compute a 'symmetrizer' $S = S^T$ with $SA = (SA)^T = A^T S$ has been a daunting task. A symmetric matrix factorization was first achieved computationally in [3]. Trying to compute well conditioned or even nonsingular matrix symmetrizers leads to an open nonlinear optimization problems over linear spaces.

I elaborate on three different classes of algorithms:

(1) The first successful approach from 2013 is *iterative* and adapts the iterative method of Huang and Nong's [2] for solving linear systems to our problem with $F = R$ or C: Find a nonsingular symmetric S so that $S \cdot A - A^T \cdot S = O_n$ [3].

(2) The second set of successful algorithms of [1] (joint with F. Dopico) uses *eigenanalyses* of A and various ways to use elements of the Jordan structure of A stably and integrate other eigen-algorithms such as Arnoldi's method together with special matrix symmetrizing method such as Datta's method for non-decomposing Hessenbergs. In the generic, the diagonalizable random entry matrix case, eigen based methods are much faster than the linear systems method and seemingly always find nonsingular symmetrizers, but for defective matrices with nontrivial eigen- and Jordan structure they generally fail to find full rank symmetrizers, i.e., they do not factor A [1].

(3) Our third algorithm, joint with Luke Oeding, Auburn, computes the entries of $S = S^T$ *directly* from the sparse linear system $S \cdot A - A^T \cdot S = O_n$.

These failures lead to the following types of matrix optimization problems:

Open Matrix Optimization Problem:

Given a basis for a subspace of n by n matrices, how can one determine a nonsingular matrix in the subspace, and how can one find one with a small condition number?

References

[1] F. Dopico and F. Uhlig, Computing matrix symmetrizers, Part 2: new methods using eigendata and linear means; a comparison. (submitted).

[2] J. Huang and L. Nong, An iterative algorithm for solving a finite-dimensional linear operator equation $T(x) = f$ with applications. *Linear Algebra Appl.* **432** (2010), No. 5, 1176–1188.

[3] F. Uhlig, Computing matrix symmetrizers, finally possible via the Huang and Nong algorithm. *Linear Multilinear Algebra* **61** (2013), No. 7, 954–969.

Boundary Representations and the Agler Algebra Over the Tri-Disk

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Arveson introduced the notion of a *boundary representation* so as to, in the setting of operator algebras, come up with a non-commutative analogue of the Choquet boundary for uniform algebras. This is a minimal set of irreducible completely contractive representations with which one can norm the algebra (for uniform algebras these are all one dimensional and correspond to point evaluations on the Choquet boundary). Dritschel and McCullough proved that if one drops irreducibility, such representations always exist (we term them extremal), and that every completely contractive representation dilates to an extremal representation. Davidson and Kennedy later refined the proof to show that sufficiently many irreducible extremal (that is, boundary) representations exist so as to norm the algebra. The proof is existential, and while it is interesting to be able to describe the boundary representations in concrete examples, this is often very difficult to do. We discuss this problem in the context of Agler's analogue of $H^\infty(\mathbb{D}^3)$. As we show, despite being a commutative algebra, it has nontrivial boundary representations which are not point evaluations.

The Angle of an Operator and Range-Kernel Complementarity

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We show that if the angle of a bounded linear operator on a Banach space with closed range and closed sum of its range and kernel is less than π, then its range and kernel are complementary. In finite dimensions and up to scalar multiples this simple geometric property characterizes operators with complementary range and kernel. Applying our main result we get simple proofs of two known results concerning eigenvalues lying in the boundary of the numerical range. For an operator on a Hilbert space we present a sufficient condition, for range-kernel complementarity, involving the distance of the boundary of the numerical range from the origin. Concluding, we also exploit some properties of operators whose spectrum does not intersect all rays emanating from the origin.

Normal Oscillations of a Pendulum with Two Capillary Viscous Fluids

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Small motions of a three-dimensional pendulum with a cavity filled with two capillary viscous fluids is considered by author in [1]. The corresponding problem about normal motions, i.e. about solutions depending on time as $\exp(-\lambda t)$, $\lambda \in \mathbb{C}$, leads to non-self-adjoint spectral problem that has been sufficiently investigated up to now.

The study of normal oscillations is reduced to the study of the matrix operator equation in orthogonal sum of Hilbert spaces. We study properties of entries of these operator matrices under the conditions of static stability with respect to the linear approximation. For it we use some definitions and assertions of the spectral theory of non-self-adjoint operators and operator pencils (see [2]–[5]). We prove the theorem on the structure of the spectrum and prove that the property of the basis for the system of eigenvalues and associated functions. Also we find the asymptotic behavior of eigenvalues for the investigated problem.

Further, we consider the case where the condition of static stability with respect to the linear approximation is not satisfied and the operator of the potential energy has at least one negative eigenvalue. We found that eigenvalues λ are located in the left-hand half-plane on the real axis and eigenvalues can transit from the right-hand half-plane to the left-hand one only through the origin of the complex plane. The inversion of Lagrange's theorem on stability is proved.

References

[1] O. A. Dudik, Problem of a hydrodynamical system consisting of a pendulum with two capillary viscous fluids. *Proceedings of the 2015 Tbilisi International Conference on Computer Sciences and Applied Mathematics*, 2015, 200–206.

[2] N. D. Kopachevsky and S. G. Krein, Operator approach to linear problems of hydrodynamics. Vol. 2. Nonself-adjoint problems for viscous fluids. Operator Theory: Advances and Applications, 146. *Birkhäuser Verlag, Basel*, 2003.

[3] I. C. Gohberg and M. G. Kreĭn, Introduction to the theory of linear non-selfadjoint operators in Hilbert space. (Russian) *Izdat. "Nauka", Moscow*, 1965.

[4] A. S. Markus, Introduction to the spectral theory of polynomial operator pencils. (Russian) *"Shtiintsa", Kishinev*, 1986.

[5] M. S. Agranovich, B. Z. Katsenelenbaum, A. N. Sivov, and N. N. Voitovich, Generalized method of eigenoscillations in diffraction theory. Translated from the Russian manuscript by Vladimir Nazaikinskii. *WILEY-VCH Verlag Berlin GmbH, Berlin*, 1999.

Mixed Boundary Value Problems for the Laplace–Beltrami Equation

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We investigate the mixed Dirichlet–Neumann boundary value problems for the Laplace–Beltrami equation on a smooth hypersurface $\mathscr{C}$ with the smooth boundary in non-classical setting in the Bessel potential spaces $\mathbb{H}^1_p(\mathscr{C})$ for $1 < p < \infty$. To the initial BVP we apply quasilocalization and obtain model BVPs for the Laplacian. The model mixed BVP on the half plane is investigated by potential method and is reduced to an equivalent system of Mellin convolution equations in Bessel potential and Besov spaces. The symbol of the obtained system is written explicitly, which provides Fredholm properties and the index of the system. The unique solvability criteria for the initial mixed BVP in the non-classical setting is derived.

Selected Properties of Weighted Shift Operators on Directed Trees

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The class of weighted shifts on directed trees, introduced quite recently in [3], is a source of interesting and often surprising examples of operators with properties not satisfied by operators from other known classes. These operators are natural research objects and are intensively studied.

There is no simple and efficient criterion that could be used to verify whether a given weighted shift on a directed tree is subnormal. We provide such a criterion in the case of bounded weighted shifts on locally finite directed trees with a help of the Andô's construction. We also supply an algorithm for checking of when a given weighted shift is not subnormal.

Moreover, we present an example of a class of subnormal operators with a dense domain and with a property such that the domain of its n-th power is not dense. In the talk we also concentrate on a q-quasinormality property of a weighted shift on a directed tree.

The talk is based on joint work with J. Stochel and Z. J. Jabłoński.

References

[1] T. Andô, Matrices of normal extensions of subnormal operators. *Acta Sci. Math. (Szeged)* **24** (1963), 91–96.

[2] P. Budzyński, P. Dymek, Z. J. Jabłoński, and J. Stochel, Subnormal weighted shifts on directed trees and composition operators in L^2-spaces with nondensely defined powers. *Abstr. Appl. Anal. 2014*, Art. ID 791817, 6 pp.

[3] Z.J. Jabłoński, I. B. Jung, and J. Stochel, Weighted shifts on directed trees. *Mem. Amer. Math. Soc.* **216** (2012), No. 1017.

Fast Quasiseparable Algorithms for Matrix Pencils

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We study the eigenvalue algorithms for matrix pencils which related to polynomial root-finding problems. Our methods are based on the fact that the matrices in such pencils have quasiseparable structure and this structure is invariant under QR iterations. Algorithms for different classes of pencils are discussed. The results of numerical tests are presented.

Joint work with P. Boito and L. Gemignani.

Numerical Range of Adjointable Operators on a Hilebert C^*-Module and Application

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We define and investigate a new numerical range of adjointable operator on some Hilbert C^*-module over C^*-algebra. It is used to characterize positive linear maps between the algebras of adjointable operators.

Rank-Deficient Spectral Factorization and Wavelets Completion Problem

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Spectral factorization is a mathematical tool which has widespread practical applications in Systems Theory, Control Engineering, Telecommunications, etc. It is the process by which a positive matrix-valued function S (defined on the unit circle $\mathbb{T}$ in the complex plane $\mathbb{C}$) is expressed in the form $S(t) = S_+(t)S_+^*(t)$. Here S_+ is a certain analytic inside $\mathbb{T}$ matrix-valued function and S_+^* is its Hermitian conjugate. The polynomial matrix spectral factorization theorem asserts that if

$$P(z) = \sum_{n=-N}^{N} C_n z^n, \quad C_n \in \mathbb{C}^{m\times m},$$

is an $m \times m$ matrix function which is positive definite for almost every $z \in \mathbb{T}$, then it admits a factorization

$$P(z) = P_+(z)P_-(z) = \sum_{n=0}^{N} A_n z^n \cdot \sum_{n=0}^{N} A_n^* z^{-n}, \quad z \in \mathbb{C}\backslash\{0\},$$

where P_+ is an $m\times m$ polynomial matrix function which is nonsingular inside $\mathbb{T}$, $\det\ P_+(z) \neq 0$ when $|z| < 1$, and $P_- = \widetilde{P_+}$ is its adjoint, $A_n^* = \overline{A_n}^T$.

Many different proofs of this important theorem appeared in the literature since the original proof in 1958. We provide a simple constructive proof of this theorem which relies only on elementary complex analysis and linear algebra and which also covers the so called rank-deficient case:

Theorem 1. *Let P be an $m \times m$ Laurent polynomial matrix function of order N which is positive semi-definite on $\mathbb{T}$ and has rank $k \leqslant n$ at almost every point. Then there exists a unique (up to a $k \times k$ right unitary factor) $m \times k$ matrix polynomial P_+ of the same order N such that $P(z) = P_+(z)P_-(z)$ holds and* $\mathrm{rank} P_+(z) = k$ *for each z inside $\mathbb{T}$.*

Theorem 1 provides a direct solution to the so called wavelet completion problem of finding matched *high pass filters* for a given *low pass filter*. Recall that a polynomial matrix function

$$W(z) = \sum_{n=0}^{N} \rho_n z^n = \left[w_{ij}(z)\right]_{i,j=\overline{1m}}, \quad \rho_n \in \mathbb{C}^{m\times m},$$

is a *wavelet matrix* if $W(z)\widetilde{W}(z) = I_m$.

Theorem 2. *For any polynomial vector function*

$$W_1(z) = \left[w_{11}(z), w_{12}(z), \ldots, w_{1m}(z)\right], \tag{1}$$

$w_{1j}(z) = \sum_{n=0}^{N} \alpha_{jn} z^n$, $j = 1, 2, \ldots, m$, *which is of the unit norm on $\mathbb{T}$, there exists a unique (up to a constant left multiplier of the block matrix form $\begin{pmatrix} 1 & 0 \\ 0 & U \end{pmatrix}$, where U is an $(m-1)\times(m-1)$ unitary matrix) $m \times m$ wavelet matrix of order and degree N ($\det W(z) = c \cdot z^N$) whose first row coincides with* (1).

Separation of Singularities, Generation of Algebras and Complete K-Spectral Sets

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In this talk, we will show a certain relation between the generation of uniform analytic algebras and complete K-spectral sets of Hilbert space operators.

Havin and Nersessian showed that, under certain geometric conditions on domains Ω_1, Ω_2 in $\mathbb{C}$, every function $f \in H^\infty(\Omega_1 \cap \Omega_2)$ can be written as $f = f_1 + f_2$, with $f_j \in H^\infty(\Omega_j)$. This result can be seen as a separation of singularities. Their result and techniques were used in our previous work to study the question of whether the collection of functions of the form $g \circ \varphi_j$, where $\{\varphi_1, \dots, \varphi_n\}$ are fixed functions from Ω into $\mathbb{D}$, generates the algebra $H^\infty(\Omega)$ (or the algebra $A(\overline{\Omega})$).

After explaining these results, we apply them to studying complete K-spectral sets. Let T be an operator on a Hilbert space H. A compact subset X of $\mathbb{C}$ is said to be a complete K-spectral set for T if $\|f(T)\|_{\mathscr{B}(H\otimes\mathbb{C}^s)} \leqslant K \sup\limits_{z\in X} \|f(z)\|_{\mathscr{B}(\mathbb{C}^s)}$, for every $s \times s$ rational matrix function f with poles outside of X of any size $s \geqslant 1$. Complete K-spectrality of an operator T in the closed unit disc $\overline{\mathbb{D}}$ is equivalent to the similarity of T to a contraction. An analogous result holds for any good simply connected domain, which involves the Riemann map $\overline{\mathbb{D}} \to \overline{\Omega}$. We will use our results on algebra generation to give tests for complete K-spectrality. These will have the form: "if $\|\varphi_k(T)\| \leqslant 1$ for every k, then $\overline{\Omega}$ is a complete K-spectral set for T, for some K." We generalize previous theorems of Badea, Beckermann, Crouzeix, B. Delyon, F. Delyon, Kazas, Kelley, Mascioni, Putinar, Sandberg, and others.

We generalize a result of Delyon and Delyon that says that every convex set containing the numerical range of an operator is a complete K-spectral set for this operator. We show how to apply this last result to obtain new criteria for similarity to a normal operator.

Acknowledgement

This is joint work with Dmitry Yakubovich (Univ. Autónoma de Madrid) and partially joint work with Michael Dritschel (Newcastle Univ.).

Wavelets on Higher-Rank Graphs

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One of the main motivations of this paper was to determine to what extent we could find faithful representations of higher-rank graph algebras on Hilbert spaces associated to a certain probability measure, and if this were possible, could we then analyze both the fractal nature of these probability spaces in terms of generalized semi-branching systems, as well as possibly give different methods of constructing generalized wavelets and KMS states on the graph algebras involved. In particular, because the generalized systems of Robertson and Steger used a finite number of irreducible commuting matrices with a common Perron–Frobenius eigenvector, our initial thought was that the Robertson–Steger algebras would be the ideal algebras for which we would be able construct representations on probability spaces associated to generalized semibranching systems of partially defined transformations.

However, upon further reflection and after undertaking an analysis of the relevant literature, we discovered that the Perron–Frobenius measure, although a useful and canonical example of a probability measure, was not strictly speaking necessary. Whenever one can construct a probability measure on a measure space equipped with what we call a "Λ-branching system", of partially defined measurable maps (which in typical examples are shift operators) for which the associated Radon–Nikodkym derivatives are almost everywhere non-zero on their domains we can construct a faithful representation of our higher-rank graph algebra. This allows us to construct representations of the higher-rank graph algebras on Hilbert spaces coming from measure spaces with a variety of geometric interpretations, that may be of independent interest.

This is joint work with E. Gillaspy, S. Kang, and J. Packer.

The Behaviour of Solutions of Degenerate Elliptic-Parabolic Equations

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Let R^{n+1} be an $(n+1)-$ dimensional Euclidian space of the points $(x,t) = (x_1,\ldots,x_n,t)$, $Q_T = \Omega \times (0,T)$ be a cylindrical domain in R^{n+1}, where Ω is a bounded n-dimensional domain with the boundary $\partial\Omega$ and $T \in (0,\infty)$. Let $Q_0 = \{(x,t) : \;\; x \in \Omega, \; t = 0\}$ and $\Gamma(Q_T) =$

$Q_0 \cup (\partial\Omega \times [0,T])$ be a parabolic boundary of Q_T. Consider the following second order degenerated elliptic-parabolic equation in Q_T

$$L_u = \sum_{i,j=1}^{n} a_{ij}(x,t)u_{x_i x_j} + \sum_{i=1}^{n} b_i(x,t)u_{x_i} + \psi(x,t)u_{tt} + C(x,t)u_t = f(x,t), \tag{1}$$

$$u\big|_{\Pi(Q_T)} = 0 \tag{2}$$

in assumption that $(a_{ij}(x,t))$ is a real symmetric matrix where for all $(x,t) \in Q_T$ and any n-dimensional vector ξ the following conditions are fulfilled

$$\gamma\omega(x)|\xi|^2 \leqslant \sum_{i,j=1}^{n} a_{ij}(x,t)\xi_i\xi_j \leqslant \gamma^{-1}\omega(x)|\xi|^2, \tag{3}$$

where $\gamma \in (0,1]$-const

$$c(x,t) \leqslant 0, \quad c(x,t) \in L_{n+1}(Q_T), \quad \Big(\sum_{i=1}^{n} b_i^2(x,t)^{\frac{1}{2}} \in L_{2(n+1)} \Big). \tag{4}$$

We determine the function $\psi(x,t) = \omega(x)w(t)\varphi\,(T-t)$, where $\omega(x)$ nonnegative function satisfy Makenxoupt condition, w, φ are continuous, non-negative and non-decreasing functions of their arguments. In the above mentioned conditions the coercive estimate is proved for strong solutions of (1), (2). Further, the weighted space $W_{2,\psi}^{2,2}(Q_T)$ is introduced and strong solvability is proved in the following form.

Theorem. *Let the conditions* (3), (4) *be fulfilled. Then at* $T \leqslant T_0$ *the problem* (1), (2) *has a unique strong solution in the space* $\overset{0}{W}{}_{2,\psi}^{2,2}(Q_T)$ *for any* $f(x,t) \in L_2(Q_T)$.

CMV Matrices in Polynomial Rootfinding

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In this talk we present some polynomial rootfinding algorithms based on matrix eigenvalue methods applied to generalized companion matrices represented as rank–one modifications of unitary matrices in CMV form. We discuss the theoretical and computational features of the proposed algorithms compared with the classical approach employing the Hessenberg form. We also describe an application to computing the zeros of an analytic function inside the unit circle in the complex plane.

The talk is partly based on joint work with R. Bevilacqua and G. Del Corso.

Some Applications of Sum of Squares Representations of Even Symmetric Forms

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In 1888, Hilbert gave a complete characterisation of the pairs $(n, 2d)$ for which a n-ary $2d$-ic form non-negative on $\mathbb{R}^n$ can be written as sums of squares of other forms, namely $\mathscr{P}_{n,2d} = \Sigma_{n,2d}$ if and only if $n = 2$, $d = 1$, or $(n, 2d) = (3, 4)$, where $\mathscr{P}_{n,2d}$ and $\Sigma_{n,2d}$ are respectively the cones of positive semidefinite (psd) and sum of squares (sos) forms (real homogenous polynomials) of degree $2d$ in n variables. This talk presents our analogue of Hilbert's characterisation under the additional assumptions of even symmetry on the given form, and few applications of sos representations of even symmetric forms.

We show that for the pairs $(n, 2d) = (3, 2d)_{d\geqslant 6}$, $(n, 8)_{n\geqslant 5}$ and $(n, 2d)_{n\geqslant 4,\, d\geqslant 7}$ there are even symmetric psd not sos n-ary $2d$-ic forms. Moreover, assuming the existence of even symmetric psd not sos n-ary decics and dodecics for $n \geqslant 4$, we establish that an even symmetric n-ary $2d$-ic psd form is sos if and only if $n = 2$ or $d = 1$ or $(n, 2d) = (n, 4)_{n\geqslant 3}$ or $(n, 2d) = (3, 8)$.

We then give necessary and sufficient conditions for an even symmetric sos form to be a sum of binomial squares (sobs) for the pairs $(n, 2)$, $(2, 2d)_{d=2,3}$, $(n, 4)_{n\geqslant 3}$ (using Ghasemi–Marshall's coefficient tests) and show that for the pairs $(2, 2d)_{d\geqslant 4}$, $(3, 8)$ there exists even symmetric sos forms that are not sobs. Finally we interpret our results on even symmetric psd forms not being sos in terms of preorderings.

This talk is partially based on joint work with S. Kuhlmann and B. Reznick.

When are the Lengths of Elementary Operators Uniformly Bounded?

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An attractive and fairly large class of linear mappings on unital C^*-algebras A are the *elementary operators*, i.e. mappings $\phi : A \to A$ which can be written as

$$\phi(x) = \sum_{i=1}^{n} a_i x b_i.$$

for some non-negative integer n and $a_i, b_i \in A$. The smallest such possible n is called the *length* of the elementary operator ϕ. In this talk I will consider the problem of characterizing those unital C^*-algebras A for which the lengths of elementary operators on A are uniformly bounded.

Poletsky Stessin Hardy Spaces in the Plane

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Recently weighted Hardy spaces were introduced by E. A. Poletsky and M. Stessin to generalize the notion of the classical Hardy spaces to more general hyperconvex domains and then to study the composition operators generated by the holomorphic mappings between such domains. In this talk we extend the classical result of Beurling which describes the invariant subspaces of the shift operator on these spaces in the plane. Joint work with Muhammed Ali Alan.

References

[1] E. A. Poletsky and M. I. Stessin, Hardy and Bergman spaces on hyperconvex domains and their composition operators. *Indiana Univ. Math. J.* **57** (2008), No. 5, 2153–2201.

[2] M. A. Alan and N. G. Göğüş, Poletsky–Stessin–Hardy spaces in the plane. *Complex Anal. Oper. Theory* **8** (2014), No. 5, 975–990.

Measurability and Weights on von Neumann Algebras

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We show that measurability of a positive operator h affiliated with a semifinite von Neumann algebra with faithful normal semifinite trace τ may be expressed by properties of the weight $\varphi = \tau(h\cdot)$ on the algebra. This is continuation of the joint work with A. Paszkiewicz [1].

References

[1] S. Goldstein and A. Paszkiewicz, Infinite measures on von Neumann algebras. *Int. J. Theor. Phys.*, 2015, doi:10.1007/s10773-015-2546-x.

On the Extension of the Toeplitz Algebra

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In his well-known work [1], L. A. Coburn showed that all isometric representations of the semigroup $\mathbb{Z}_+$ generate C^*-algebras which are canonically isomorphic to the Toeplitz algebra. Many authors have generalized this result to a wider class of semigroups. R. G. Douglas [2] showed that all non-unitary isometric representations of an arbitrary positive cone of real numbers $\mathbb{R}$ generate canonically isomorphic algebras. G. J. Murphy [3] prowed a similar result for positive cones of totally ordered Abelian groups. On the other hand, G. J. Murphy [3] and S. Y. Jang [4] have shown that this theorem is not true for the semigroup $\mathbb{Z}_+\backslash\{1\}$. Isometric representations of the semigroup $\mathbb{Z}_+\backslash\{1\}$ were investigated in the works [5], [6].

In this talk we introduce the notion of π-extension of the semigroup $\mathbb{Z}_+$. We study properties of C^*-algebras generated by the π-extension of the semigroup $\mathbb{Z}_+$. Also the concept of the inverse π-extension of semigroup $\mathbb{Z}_+$ is introduced. We show that if π is an irreducible representation of the semigroup $\mathbb{Z}_+$, then $\mathbb{Z}_+$ has no non-trivial inverse π-extension. In the case when π is a reducible representation, there exists a non-inverse π-extension. On the other hand, we show that for any isometric representation of $\mathbb{Z}_+$ there always exists a non-inverse π-extension. Next, we study the extensions of the Toeplitz algebra generated by the inverse π-extensions of the semigroup $\mathbb{Z}_+$. It is proved that among such extensions there are extensions which are the tower of the nested Toeplitz algebras inductive limit of which is the C^*-algebra considered in the work of Douglas [2].

The talk is based on joint work with the co-authors T. A. Grigoryan and E. V. Lipacheva: *"On the extension of the Toeplitz algebra by isometries". The Varied Landscape of Operator Theory, 137–146, Theta, 2014*.

References

[1] L. A. Coburn, The C^*-algebra generated by an isometry. *Bull. Amer. Math. Soc.* **73** (1967), 722–726.

[2] R. G. Douglas, On the C^*-algebra of a one-parameter semigroup of isometries. *Acta Math.* **128** (1972), No. 3-4, 143–151.

[3] G. J. Murphy, Ordered groups and Toeplitz algebras. *J. Operator Theory* **18** (1987), No. 2, 303–326.

[4] S. Y. Jang, Generalized Toeplitz algebra of a certain non-amenable semigroup. *Bull. Korean Math. Soc.* **43** (2006), No. 2, 333–341.

[5] I. Raeburn and S. T. Vittadello, The isometric representation theory of a perforated semigroup. *J. Operator Theory* **62** (2009), No. 2, 357–370.

[6] V. A. Tepoyan, On isometric representations of the semigroup $\mathbb{Z}_+\setminus\{1\}$. (Russian) *Izv. Nats. Akad. Nauk Armenii Mat.* **48** (2013), No. 2, 51–62; translation in *J. Contemp. Math. Anal.* **48** (2013), No. 2, 78–84.

Solition Theory of the KdV Equation and Hankel Operators with Oscillating Symbols

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Solition theory and the theory of Hankel (and Toeplitz) operators have stayed essentially hermetic to each other. This talk is concerned with linking together these two very active and extremely large theories. On the prototypical example of the Cauchy problem for the Korteweg-de Vries (KdV) equation we demonstrate the power of the language of Hankel operators in which symbols are conveniently represented in terms of the scattering data for the Schrodinger operator associated with the initial data for the KdV equation. This approach yields short-cuts to already known results as well as to a variety of new ones (e.g. wellposedness beyond standard assumptions on the initial data) which are achieved by employing some subtle results for Hankel operators.

Reduced Differential Transform Method for Partial Differential Equations

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In this paper, we present a numerical method called "Reduced Differential Transform Method" to solve linear and nonlinear partial differential equations which was introduced by Keskin and Oturanc [1] and also Keskin's Ph.D. Thesis (2010) [2]. Some of linear and nonlinear problems are considered to solve by Reduced Differential Transform Method.

Reduced Differential Transform Method which has an alternative approach of problems is presented to overcome the demerit of complex calculation, discretization, linearization or small perturbations of well-known numerical and analytical methods such as Adomian Decomposition, Differential Transform, Homotopy Perturbation, Variational Iteration etc. And also the main advantage of this method is providing to its user with an analytical approximation, in many cases an exact solution, in a rapidly convergent sequence with elegantly computed terms [1, 3, 4]. Recently, some of research articles related to Reduced Differential Transform Method have been published in literature as [5–9].

References

[1] Y. Keskin, G. Oturanç: Reduced Differential Transform Method for Partial Differential Equations, *Int. J. Nonlinear Sci. Numer. Simul.* **10** (2009), No. 6, 741-7509; doi: 10.1515/IJNSNS.2009.10.6.741.

[2] Y. Keskin, Ph.D. thesis. (Turkish) *Selcuk University*, 2010.

[3] Y. Keskin and G. Oturanç, Reduced differential transform method for solving linear and nonlinear wave equations. *Iran. J. Sci. Technol. Trans. A Sci.* **34** (2010), No. 2, 113–122.

[4] Y. Keskin and G. Oturanç, Reduced differential transform method for fractional partial differential equations. *Nonlinear Science Letters A* **1(2)** (2010), 61–72.

[5] R. Abazari and M. Abazari, Numerical simulation of generalized Hirota–Satsuma coupled KdV equation by RDTM and comparison with DTM. *Commun. Nonlinear Sci. Numer. Simul.* **17** (2012), No. 2, 619–629.

[6] P. K. Gupta, Approximate analytical solutions of fractional Benney–Lin equation by reduced differential transform method and the homotopy perturbation method. *Comput. Math. Appl.* **61** (2011), No. 9, 2829–2842.

[7] M. Sohail and S. T. Mohyud-Din, Reduced differential transform method for Laplace equations. *Internat. J. Modern Theoret. Phys.* **1** (2012), No. 2, 6–12.

[8] M. A. Abdou and A. Yildirim, Approximate analytical solution to time fractional nonlinear evolution equations. *Internat. J. Numer. Methods Heat Fluid Flow* **22** (2012), No. 6-7, 829–838.

[9] R. Abazari and B. Soltanalizadeh, Reduced differential transform method and its application on Kawahara equations. *Thai J. Math.* **11** (2013), No. 1, 199–216.

[10] M. Gubes, Ph.D. Thesis. (Turkish) *Selcuk University, Konya*, 2015 (to appear).

Blow-Up Solutions Some Classes of the Nonlinear Parabolic Equations

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In this paper the unbounded increasing solutions of the nonlinear parabolic equations for the finite time is investigated. Before we considered Dirichlet boundary condition. In this paper Neuman boundary problem is investigated.

The sufficient condition for nonlinearity is established. Under this condition every solution of the investigated problem is blown-up. I.e., there is number $T > 0$ such that

$$\|u(x;t)\|_{L_2(R^n)} \to \infty, \quad t \to T < \infty.$$

The existence of the solution is proved by smallness of the initial function.

These type of nonlinear equations describe the processes of electron and ionic heat conductivity in plasma, fusion of neutrons and etc. One of the essential ideas in theory of evolutional equations is known as method of eigenfunctions. In this paper we apply such method. We different boundary problem is considered.

In [1] the existence of unbounded solution for finite time with a simple nonlinearity have been proved. In [2] has been shown that, under the critical exponent any nonnegative solution is unbounded increasing for the finite time. Similar results were obtained in [3] and corresponding theorems are called Fujita–Hayakawa's theorems. More detailed reviews can be found in [4-6].

References

[1] S. Kaplan, On the growth of solutions of quasi-linear parabolic equations. *Comm. Pure Appl. Math.* **16** (1963), 305–330.

[2] H. Fujita, On the blowing up of solutions of the Cauchy problem for $u_t = \Delta u + u^{1+\alpha}$. *J. Fac. Sci. Univ. Tokyo Sect. I* **13** (1966), 109–124).

[3] K. Hayakawa, On nonexistence of global solutions of some semilinear parabolic differential equations. *Proc. Japan Acad.* **49** (1973), 503–505.

[4] V. A. Galaktionov and H. A. Levine, A general approach to critical Fujita exponents in nonlinear parabolic problems. *Nonlinear Anal.* **34** (1998), No. 7, 1005–1027.

[5] K. Deng and H. A. Levine, The role of critical exponents in blow-up theorems: the sequel. *J. Math. Anal. Appl.* **243** (2000), No. 1, 85–126.

[6] A. A. Samarskii, V. A. Galaktionov, S. P. Kurdyumov, and A. P. Mikhailov, Blow-up in quasi linear parabolic equations. (Russian) *Nauka, Moscow*, 1987.

Bilinear Multipliers of Some Functional Spaces

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The bounded function $m(\xi,\eta)$ defined on $\mathbb{R}^n \times \mathbb{R}^n$ is said to be bilinear multiplier on $\mathbb{R}^n$ of type (p_1,p_2,p_3) if

$$B_m(f,g)(x) = \int\limits_{\mathbb{R}^n}\int\limits_{\mathbb{R}^n} \widehat{f}(\xi)\widehat{g}(\eta)m(\xi,\eta)e^{2\pi i\langle \xi+\eta,x\rangle}\,d\xi\,d\eta,$$

defines a bounded bilinear operator from $L^{p_1}(\mathbb{R}^n) \times L^{p_2}(\mathbb{R}^n)$ to $L^{p_3}(\mathbb{R}^n)$. The study of bilinear multipliers goes back to the work Coifman and Meyer in [2]. A non-trivial example is given by $m(x) = -i\,\mathrm{sgn}(x)$ which leads to the bilinear Hilbert transform. Kulak and Gürkanlı extended these results to weighted Lebesgue and variable exponent Lebesgue spaces [4].

Let $1 \leqslant p,q < \infty$ and w,ω be weight functions on $\mathbb{R}^n$. In [3] a functional space $A^{p,q}_{w,\omega}(\mathbb{R}^n)$ defined and investigated by Fischer, Gürkanlıand Liu. It is generalized to the variable exponents case $A^{p(x)}_{\omega}(\mathbb{R}^n)$ in Aydın, and Gürkanlı, in [1].

Let $1 \leqslant p_1,p_2,q_1,q_2 < \infty$ and $w_1,w_2,\omega_1,\omega_2$ be weight functions on $\mathbb{R}^n$. In the present paper in first section we define the bilinear multiplier B_m from $A^{p_1,q_1}_{w_1,\omega_1}(\mathbb{R}^n) \times A^{p_2,q_2}_{w_2,\omega_2}(\mathbb{R}^n)$ to

$A^{p_3,q_3}_{w_3,\omega_3}(\mathbb{R}^n)$, and discuss that under what conditions B_m is bounded. Later we investigate the properties of the space of bilinear multipliers and give some examples. In the second section similarly we define the bilinear multiplier B_m from $A^{p_1(x)}_{\omega_1}(\mathbb{R}^n) \times A^{p_2(x)}_{\omega_2}(\mathbb{R}^n)$ to $A^{p_3(x)}_{\omega_3}(\mathbb{R}^n)$. At the end of this section we investigate the properties of the space of bilinear multipliers.

References

[1] İ Aydın and A. T. Gürkanlı, Weighted variable exponent amalgam spaces $W(L^{p(x)}, L^q_w)$. *Glas. Mat. Ser. III* **47(67)** (2012), No. 1, 165–174.

[2] R. R. Coifman and Y. Meyer, Fourier analysis of multilinear convolutions, Calderón's theorem, and analysis of Lipschitz curves. *Euclidean harmonic analysis (Proc. Sem., Univ. Maryland, College Park, Md., 1979)*, pp. 104–122, Lecture Notes in Math., 779, *Springer, Berlin*, 1980.

[3] R. H. Fischer, A. T Gürkanlıand T. S. Liu, On a family of weighted spaces. *Math. Slovaca* **46** (1996), No. 1, 71–82.

[4] Ö. Kulak and A. T. Gürkanlı, Bilinear multipliers of weighted Lebesgue spaces and variable exponent Lebesgue spaces. *J. Inequal. Appl.* **2013**, 2013:259, 21 pp.

On the Spectrum of Certain Random Operators: A Link to Julia Sets

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After the introduction of random matrices to nuclear physics by Eugene Wigner in 1955, random quantum systems have grown in popularity. Wigner's idea was to consider families of Hamiltonians that underlie a certain probability distribution to describe overly complicated systems. Of particular interest are, of course, the spectra of these Hamiltonians. In this talk we consider random, in general non-self-adjoint, tridiagonal operators on the Hilbert space of square-summable sequences. To model randomness, we use an approach by Davies that eliminates all probabilistic arguments.

Despite the rising interest, not much is known about the spectra of non-self-adjoint random operators. The Feinberg–Zee random hopping matrix reveals this in a beautiful manner. The boundary of its spectrum appears to be fractal, but a proof has not been found yet. While we can not give a proof either, we present a reason why this is very plausible. Certain tridiagonal operators share remarkable symmetries that allow us to enlarge known subsets of the spectrum by sizeable amounts. In some cases like the Feinberg–Zee random hopping matrix, this implies that the spectrum contains an infinite sequence of Julia sets.

References

[1] R. Hagger, Symmetries of the Feinberg–Zee Random Hopping Matrix. http://arxiv.org/abs/1412.1937.

Definite Quadratic Forms and Hyperbolic Polynomials

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Well known properties of real symmetric matrices, such as realness of its eigenvalues, or the stronger spectral theorem, are essentially and quite explicitly based on the fact that the standard inner product on $\mathbb{R}^n$ is definite and therefore anisotropic.

These properties make it obvious that a polynomial $F \in \mathbb{R}[x, y, z]_n$ that admits a definite determinantal representation is hyperbolic. That means if there are real symmetric matrices $A, B, C \in \mathbb{R}^{n\times n}$ and $v \in \mathbb{R}^3$ such that

$$F = \det(Ax + By + Cz) \tag{$*$}$$

and $Av_1 + Bv_2 + Cv_3$ definite, then F is hyperbolic with respect to v, i.e. for all $x \in \mathbb{R}^3$ the roots of the univariate polynomial

$$F(tv - x) \in \mathbb{R}[t]$$

are all real.

It was conjectured by Peter Lax in 1958 [1], that the converse to this observation also holds. The affirmative answer was given by Helton and Vinnikov about ten years ago in [2].

In the talk we want to give an idea on how the above mentioned anisotropy also greatly simplifies the study of definite representations of the form $(*)$ as opposed to indefinite ones. An algebraic and more elementary proof of the Helton-Vinnikov Theorem can be found in [3].

References

[1] P. D. Lax, Differential equations, difference equations and matrix theory. *Comm. Pure Appl. Math.* **11** (1958), 175–194.

[2] J. W. Helton and V. Vinnikov, Linear matrix inequality representation of sets. *Comm. Pure Appl. Math.* **60** (2007), No. 5, 654–674.

[3] C. Hanselka, Definite determinantal representations of ternary hyperbolic forms. *Preprint*, `http://arxiv.org/pdf/1411.1661.pdf`.

The Eigenvalues Function and the Inverse Sturm–Liouville Problems

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Let $\mu_n = \mu_n(q, \alpha, \beta)$, $n = 0, 1, 2, \dots$, are the eigenvalues (enumerated in the increasing order) of the self-adjoint operator $L(q, \alpha, \beta)$, corresponding to the Sturm–Liouville problem

$$\begin{gathered} -y'' + q(x)y = \mu y, \quad x \in (0, \pi), \quad q \in L^1_{\mathbb{R}}[0, \pi], \quad \mu \in \mathbb{C}, \\ y(0) \cos \alpha + y'(0) \sin \alpha = 0, \quad \alpha \in (0, \pi], \\ y(\pi) \cos \beta + y'(\pi) \sin \beta = 0, \quad \beta \in [0, \pi). \end{gathered}$$

Definition. The function μ of two variables, defined on $(0, \infty) \times (-\infty, \pi)$ by formula

$$\mu(\alpha + \pi k, \beta - \pi m) \overset{def}{=} \mu_{k+m}(q, \alpha, \beta), \quad k, m = 0, 1, 2, \dots, \tag{1}$$

called the eigenvalues function (EVF) of the family of operators $\{L(q, \alpha, \beta), \ \alpha \in (0, \pi], \ \beta \in [0, \pi)\}$. Thus, every function q from $L^1_{\mathbb{R}}[0, \pi]$ we map into a function μ, given by formula (1). We investigate the property of this function and we prove that map $q \to \mu$ is one-to-one. In particular we give the algorithm of construction q by function μ, which have the property of EVF.

Phenomena of Projectivity and Freeness in Classical and Quantum Functional Analysis

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The concept of the projectivity, together with its dual version (injectivity) and and a weaker version (flatness) is extremely important in algebra. The variants of this concept, now established in operator theory, in particular, in representation theory of "classical" and "quantum" Banach algebras, are also of considerable importance. In analysis, however, there exist several different approaches to this concept, corresponding to different problems of lifting of operators.

We shall discuss several (comparatively rigid and comparatively tolerant) variants of projectivity in operator theory and show that all of them can be included in a certain general scheme. This scheme allows to study projectivity by means of the so-called freeness. The relevant free objects are defined by the same way as free groups, free modules, free Banach spaces etc. Projective objects are direct summands (in a proper sense) of free objects.

We shall describe free objects, corresponding to each of the discussed versions of the projectivity. In particular, we shall characterize free operator spaces in terms of spaces of nuclear operators. In the "classical" context we shall characterize metrically free normed spaces: they turn out to be subspaces in $l_1(\Lambda)$, consisting of functions with finite supports. As a corollary, all metrically projective normed spaces are free.

On Wiener–Hopf and Mellin Operators Arising in the Theory of Lévy Processes

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Markov processes are well understood in the case when they take place in the whole Euclidean space. However, the situation becomes much more complicated if a Markov process is restricted to a domain with a boundary, and then a satisfactory theory only exists for processes with continuous trajectories. The aim of this talk is to present interim results of an ongoing research project concerned with a symmetric stable Lévy process defined on a half-line. After some analysis, the problem is reformulated in the context of an algebra of multiplication, Wiener-Hopf and Mellin operators, and the nature and significance of the resulting symbol, see [1], is examined.

References

[1] R. Duduchava, On algebras generated by convolutions and discontinuous functions. Special issue: Wiener–Hopf problems and applications (Oberwolfach, 1986). *Integral Equations Operator Theory* **10** (1987), No. 4, 505–530.

Inverse Scattering for Singular Energy-Dependent Schrödinger Equations

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We discuss the direct and inverse scattering theory for one-dimensional energy-dependent Schrödinger equations

$$-y'' + q(x)y + 2kp(x)y = k^2y \tag{5}$$

on the half-line under minimal assumptions on the real-valued potentials p and q. Namely, the potential q is a distribution from the space $H^{-1}_{2,loc}(\mathbb{R}_+)$ enjoying certain integrability conditions, while p belongs to $L_1(\mathbb{R}_+) \cap L_2(\mathbb{R}_+)$. The above equation (5), being subject to some boundary conditions, leads to a non-standard spectral problem involving both the spectral parameter k and its square k^2. Due to this the spectrum of the problem is no longer real and may contain pairs of complex conjugate eigenvalues.

In the talk, we introduce the scattering data for (5) and give a complete description of such data for the considered class of problems. Also, a procedure reconstructing the potentials p and q from the scattering data is described and continuity of the mapping between the potentials p and q and the corresponding scattering data is established.

The talk is based on a joint project with S. Manko (Lviv, Ukraine).

Subnormality of Composition Operators: Exotic Examples. II

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In my talk I will give an affirmative answer to the question whether there exists a non-hyponormal composition operator in an L^2-space generating Stieltjes moment sequences over a locally finite directed graph. The constructive method of solving the problem partly based on transforming the Krein and the Friedrichs measures coming either from Al-Salam–Carlitz q-polynomials or from a quartic birth and death process.

This is the second part of a two-part talk. The first part will be presented by P. Budzyński in the same Thematic Session.

The talk is based on joint work with P. Budzyński, I. B. Jung, and J. Stochel (cf. [1]).

References

[1] P. Budzyński, Z. J. Jabłoński, I. B. Jung and J. Stochel, Subnormality of unbounded composition operators over one-circuit directed graphs: exotic examples. *Preprint*, 2015.

Some Divergence Integrals and Dirac Delta Function

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We have proven in [1] the following

Theorem. For any real τ one can write:

$$\int_0^1 t^{i\tau-1} dt = 2\pi\delta(\tau). \tag{1}$$

Using the equation (1) one can show that

$$\int_0^1 t^{i\tau-1} \ln(t)\, dt = -2\pi i\delta'(\tau). \tag{2}$$

where the derivative of the δ function is determined as follows (see, e.g., [2, Chapter 1]):

$$\int_{-a}^{a} dx\delta'(x)\phi(x) = -\int_{-a}^{a} dx\, \delta(x)\phi'(x), \quad \phi(x) \in C_1, \quad x \in (-a, a).$$

From the formula (2) one obtains:

$$\int_{0}^{x} \tau\, d\tau \int_{0}^{1} du\, u^{i\tau-1} \ln(u) = 2\pi i\theta(x),$$

where $\theta(x)$ is the Heaviside unit function derivative of that may be expressed by the δ function (see, e.g., [2, Chapter 1]):

$$\theta'(x) = \delta(x).$$

For any function $\phi(x)$ $(\phi(x) \in C_1, x \in (-a, a))$, from the formula (2) we get

$$\int_{-a}^{a} d\tau\, \phi(\tau) \int_{0}^{1} dt\, t^{i\tau-1} \ln(t) = 2\pi i \int_{-a}^{a} d\tau\, \delta(\tau)\phi'(\tau) = 2\pi i\phi'(0).$$

For example:

$$\int_{-a}^{a} d\tau\, \sin(\tau) \int_{0}^{1} dt\, t^{i\tau-1} \ln(t) = 2\pi i.$$

The results obtained can be successfully used in some physical calculations, namely, in the quantum mechanical problem of two charged particles of continuous spectrum, in some topics of special functions theory, etc.

References

[1] V. Jikia and I. Lomidze, On connection between the Euler beta function and the Dirac delta function. *Proc. Georgian Acad. Sci., Math.* (to be published).

[2] I. M. Gel'fand and G. E. Shilov, Generalized functions. Vol. I: Properties and operations. Translated by Eugene Saletan, *Academic Press, New York–London*, 1964.

A Construction of Pro-C^*-Algebras from Pro-C^*-Correspondences

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A C^*-correspondence is a natural generalization of a Hilbert C^*-bimodule. Namely it is a pair (X, A), where X is a right Hilbert A-module together with a left action of A on X. In [5], M. V. Pimsner first showed how to associate a C^*-algebra to certain C^*-correspondences, introducing a class of C^*-algebras that are now known as Cuntz–Pimsner algebras. It was later that T. Katsura in his series of papers [2], [3], extended the former construction and associated a certain C^*-algebra to every C^*-correspondence. Katsura's more general construction includes a wide range of algebras, amongst them the crossed product of a C^*-algebra by a Hilbert C^*-bimodule.

The extension of so rich in results concepts to the case of pro-C^*-algebras could not be disregarded. A pro-C^*-algebra $A[\tau_\Gamma]$ is a complete topological $*$-algebra for which there exists a directed family of C^*-seminorms $\Gamma = \{p_\lambda : \lambda \in \Lambda\}$ defining the topology τ_Γ. In [4], N. C. Phillips considered Hilbert modules over pro-C^*-algebras and studied their structure. Subsequently, in [1] we defined and studied the crossed product of a pro-C^*-algebra by a Hilbert pro-C^*-bimodule which is a generalization of crossed products of pro-C^*-algebras by inverse limit automorphisms. All the above, gave us the impetus to generalize the important topic of C^*-correspondences in the setting of pro-C^*-algebras and to examine under which conditions we can associate a pro-C^*-algebra to a pro-C^*-correspondence.

The talk is based on joint work with co-author Ioannis Zarakas, Department of Mathematics, University of Athens, Greece.

References

[1] M. Joiţa and I. Zarakas, Crossed products by Hilbert pro-C^*-bimodules. *Studia Math.* **215** (2013), No. 2, 139–156.

[2] T. Katsura, A construction of C^*-algebras from C^*-correspondences. *Advances in quantum dynamics (South Hadley, MA, 2002)*, 173–182, Contemp. Math., 335, *Amer. Math. Soc., Providence, RI*, 2003.

[3] T. Katsura, On C^*-algebras associated with C^*-correspondences. *J. Funct. Anal.* **217** (2004), No. 2, 366–401.

[4] N. C. Phillips, Inverse limits of C^*-algebras. *J. Operator Theory* **19** (1988), No. 1, 159–195.

[5] M. V. Pimsner, A class of C^*-algebras generalizing both Cuntz–Krieger algebras and crossed products by $\mathbf{Z}$. *Free probability theory (Waterloo, ON, 1995)*, 189–212, Fields Inst. Commun., 12, *Amer. Math. Soc., Providence, RI*, 1997.

On the Distance to Normal Elements in C^*-Algebras of Real Rank Zero

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In 1993, Huaxin Lin showed that the distance in the operator norm from an $n \times n$-matrix A, $\|A\| \leqslant 1$, to the closest *normal* $n \times n$-matrix can be estimated from above by some function $F(\|[A, A^*]\|)$ such that $F(t) \to 0$ as $t \to 0$, where the function F does not depend on the dimension. The original proof didn't give any other information on the behavior of F around zero. We obtain an order sharp estimate for the distance from a given bounded operator A on a Hilbert space to the set of normal operators in terms of $\|[A, A^*]\|$ and the distance to the set of invertible operators. For finite matrices, this implies that one can choose $F(t) = Ct^{1/2}$. A slightly modified estimate holds in a general C^*-algebra of real rank zero.

The talk is based on joint results with Professor Yuri Safarov (King's College, London).

Contractive Determinantal Representations of Stable Polynomials on a Matrix Polyball

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We show that a polynomial p with no zeros on the closure of a matrix unit polyball, a.k.a. a cartesian product of Cartan domains of type I, and such that $p(0) = 1$, admits a strictly contractive determinantal representation, i.e., $p = \det(I - KZ_n)$, where $n = (n_1, \dots, n_k)$ is a k-tuple of nonnegative integers, $Z_n = \bigoplus_{r=1}^{k} (Z^{(r)} \otimes I_{n_r})$, $Z^{(r)} = [z_{ij}^{(r)}]$ are complex matrices, p is a polynomial in the matrix entries $z_{ij}^{(r)}$, and K is a strictly contractive matrix. This result is obtained via a noncommutative lifting and a theorem on the singularities of minimal noncommutative structured system realizations. The talk is based on a joint work with A. Grinshpan, V. Vinnikov, and H. J. Woerdeman.

The Structure of Crossed Product C^*-Algebras with Finite Groups

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Let A be a C^*-algebra and G be a group acting on A. The dual space of a crossed product $A \rtimes_\sigma G$ has a rich and deep structure. Recently, Echterhoff and Williams gave a concrete description of the dual space in the case of a strictly proper action on a continuous trace C^*-algebra [1]. We obtain analogous results in the case of a finite group acting on an arbitrary C^*-algebra. In particular, we construct a space $\widetilde{\Gamma}$ which consists of pairs of irreducible representations of A and irreducible projective representations of subgroups of G. We endow $\widetilde{\Gamma}$ with a topology so that the dual of $A \rtimes_\sigma G$ is homeomorphic to the orbit space $G\backslash\widetilde{\Gamma}$.

References

[1] S. Echterhoff and D. P. Williams, Structure of crossed products by strictly proper actions on continuous-trace algebras. *Trans. Amer. Math. Soc.* **366** (2014), Nno. 7, 3649–3673.

The Set of Tuples of Partial Indices of Triangular Matrix Functions

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We will discuss the problem of constructive description of the possible tuples of partial indices for triangular matrix functions whose diagonal elements are factorable with the fixed vector $\chi \in \mathbb{Z}^n$ of their indices. This set $A(\chi)$ contains the set $E(\chi)$ of all vectors obtained from χ by permuting its components and is in turn contained in the set $M(\chi)$ of all vectors majorized by χ. The conditions on χ under which $A(\chi) = E(\chi)$ or $A(\chi) = M(\chi)$ are also discussed.

Wave Diffraction by Wedges

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The problem of plane wave diffraction by a wedge sector having arbitrary aperture angle has a very long and interesting research background. In fact, we may recognize significant research on this topic for more than one century. Despite this fact, until the paper [1] no clear unified approach was implemented to treat such a problem from a rigourous mathematical way and in a consequent appropriate Sobolev space setting. In the present talk, we are considering the corresponding boundary value problems for the Helmholtz equation, with complex wave number, admitting combinations of Dirichlet and Neumann boundary conditions. The main ideas are based on a convenient combination of potential representation formulas associated with (weighted) Mellin pseudo-differential operators in appropriate Sobolev spaces, and a detailed Fredholm analysis. Thus, we prove that the problems have unique solutions (with continuous dependence on the data), which are represented by the single and double layer potentials, where the densities are solutions of derived pseudo-differential equations on the half-line.

The talk is based on joint work [1] with co-author Luís P. Castro.

References

[1] L. P. Castro and D. Kapanadze, Wave diffraction by wedges having arbitrary aperture angle. *J. Math. Anal. Appl.* **421** (2015), No. 2, 1295–1314.

Index Formula for Convolution Type Operators with Data Functions in $\mathrm{alg}(SO, PC)$

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We establish an index formula for the Fredholm convolution type operators

$$A = \sum_{k=1}^{m} a_k W^0(b_k)$$

acting on the space $L^2(\mathbb{R})$, where a_k, b_k belong to the C^*-algebra $\mathrm{alg}(SO, PC)$ of piecewise continuous functions on $\mathbb{R}$ that slowly oscillate at $\pm\infty$, that is, satisfy the condition

$$\lim_{x\to\pm\infty} \max_{t,\tau\in[x,2^{\pm 1}x]} |c(t) - c(\tau)| = 0 \text{ for } c \in \mathrm{alg}(SO, PC).$$

We assume that all functions a_k, b_k admit finite sets of discontinuities. The study is based on applying the so-called truncated operators

$$A_r = \sum_{k=1}^{m} a_{k,r} W^0(b_{k,r})$$

for all sufficiently large $r > 0$, where the functions $a_{k,r}, b_{k,r} \in PC$ are obtained from $a_k, b_k \in \mathrm{alg}(SO, PC)$ by extending their values at $\pm r$ to all $\pm t \geqslant r$, respectively. Making use of two special decompositions and then a separation of discontinuities, we prove that $\mathrm{ind}\, A = \lim_{r\to\infty} \mathrm{ind}\, A_r$ although $A = \text{s-}\lim_{r\to\infty} A_r$ only, where A_r are Fredholm operators for sufficiently large $r > 0$. Applying then Duduchava's results on indices of operators A_r with piecewise continuous data functions, we calculate the index of the operator A.

The talk is based on joint work with M. Amélia Bastos and Catarina C. Carvalho.

The Haseman Boundary Value Problem with Oscillating Matrix Coefficient

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Given $1 < p < \infty$, let $L^p_n(\mathbb{R})$ denote the Banach space of vector functions $\varphi = \{\varphi_k\}_{k=1}^n$ with entries $\varphi_k \in L^p(\mathbb{R})$ and the norm $\|\varphi\| = \big(\sum\limits_{k=1}^{n} \|\varphi_k\|^p_{L^p(\mathbb{R})}\big)^{1/p}$. Let $C_b(\mathbb{R})$ be the C^*-algebra of all bounded continuous complex-valued functions on $\mathbb{R} = (-\infty, +\infty)$, and let SO denote the C^*-algebra consisting of all functions $a \in C_b(\mathbb{R})$ that slowly oscillate at $\pm\infty$, that is, satisfy the condition

$$\lim_{x\to\pm\infty} \max_{t,\tau\in[x,2x]} |a(t) - a(\tau)| = 0.$$

Let SAP be the C^*-algebra of semi-almost periodic functions on $\mathbb{R}$, which consists of all functions in $C_b(\mathbb{R})$ that have different, in general, almost periodic asymptotics at $\pm\infty$, and let $\mathrm{alg}\,(SO, SAP)$ stand for the minimal C^*-algebra containing SO and SAP.

The talk is devoted to studying the following Haseman boundary value problem: Find a function Φ analytic in the half-planes $\mathbb{C}^\pm = \{z \in \mathbb{C} : \ \pm\mathrm{Im}\, z > 0\}$, represented by the Cauchy type integral over $\mathbb{R}$ with a density $\varphi \in L^p_n(\mathbb{R})$ and satisfying the boundary condition

$$\Phi^+[\alpha(x)] = G(x)\Phi^-(x) + g(x) \ \text{ for } \ x \in \mathbb{R},$$

where $\Phi^\pm(x)$ are angular boundary values of Φ on $\mathbb{R}$ from $\mathbb{C}^\pm$, G is an $n \times n$ matrix function with entries in $\mathrm{alg}\,(SO, SAP)$, $g \in L^p_n(\mathbb{R})$, and α is an orientation-preserving diffeomorphism of $\mathbb{R}$ onto itself such that $\alpha(\infty) = \infty$, $\log \alpha' \in C_b(\mathbb{R})$ and $\alpha' \in SO$.

The Haseman boundary value problem under different conditions on its data was studied by C. Haseman, D. A. Kveselava, L. I. Chibrikova, G. F. Mandzhavidze and B. V. Khvedelidze, I. B. Simonenko, and by the author in teams with V. G. Kravchenko, A. V. Aizenshtat and G. S. Litvinchuk. Reducing the Haseman boundary value problem by the Sokhotski–Plemelj formulas to an equivalent singular integral operator T with shift α whose derivative slowly oscillates at $\pm\infty$ and with matrix coefficient G admitting at ∞ piecewise slowly oscillating and semi-almost periodic discontinuities, and applying the limit operators techniques and results on almost periodic factorability for a family of almost periodic asymptotics related to the points in the fiber $M_\infty(SO)$ of the maximal ideal space $M(SO)$ of SO over ∞, we obtain a Fredholm criterion and an index formula for the operator T acting on the space $L^p_n(\mathbb{R})$.

The Index of Weighted Singular Integral Operators with Shifts and Slowly Oscillating Data

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Let α and β be orientation-preserving diffeomorphism (shifts) of $\mathbb{R}_+ = (0, \infty)$ onto itself with the only fixed points 0 and ∞. We establish a Fredholm criterion and calculate the index of the weighted singular integral operator with shifts

$$(aI - bU_\alpha)P_\gamma^+ + (cI - dU_\beta)P_\gamma^-,$$

acting on the space $L^p(\mathbb{R}_+)$, where $P_\gamma^\pm = (I \pm S_\gamma)/2$ are the operators associated to the weighted Cauchy singular integral operator S_γ given by

$$(S_\gamma f)(t) = \frac{1}{\pi i} \int_{\mathbb{R}_+} \left(\frac{t}{\tau}\right)^\gamma \frac{f(\tau)}{\tau - t}\, d\tau$$

with $\gamma \in \mathbb{C}$ satisfying $0 < 1/p + \Re\gamma < 1$, and U_α, U_β are the isometric shift operators given by

$$U_\alpha f = (\alpha')^{1/p}(f \circ \alpha), \quad U_\beta f = (\beta')^{1/p}(f \circ \beta),$$

under the assumptions that the coefficients a, b, c, d and the derivatives α', β' of the shifts are bounded and continuous on $\mathbb{R}_+$ and admit discontinuities of slowly oscillating type at 0 and ∞. The obtained index formula in a sense combines the index formulas for Mellin pseudodifferential operators with slowly oscillating symbols and for singular integral operators with shifts and piecewise continuous data.

The talk is based on joint work with Yuri Karlovich and Amarino Lebre.

Indefinite Metric Spaces and Operator Linear Fractional Relations

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This is a natural continuation of our previous work in the considered area.

We consider strict and bistrict plus-operators between spaces with indefinite metrics, in particular, Krein spaces (or J-spaces). We call a plus-operator T in a Krein space strict if $T = dA$, where $d > 0$ is constant and A is a J-expansion, and we call T bistrict if both T and its conjugate operator T^* are strict plus-operators.

We consider operator and geometric sufficient and necessary conditions for a given strict plus-operator T in a Krein space H to be bistrict as an operator between H and $\operatorname{Im} T$ with the induced indefinite metric.

It is well known that a plus-operator T defines an operator linear fractional relation. In particular, we consider the special case of linear-fractional transformations. In the case of Hilbert spaces $\mathfrak{H}_1$ and $\mathfrak{H}_2$, each linear-fractional transformation of the closed unit ball $\mathfrak{K}$ of the space $\mathfrak{L}(\mathfrak{H}_1, \mathfrak{H}_2)$ is of the form

$$\mathfrak{F}_T(K) = (T_{21} + T_{22}K)(T_{11} + T_{12}K)^{-1}$$

and is generated by the bistrict plus-operator T.

The theory of bistrict plus-operaters and generated linear fractional transformations forms a significant part of the theory of spaces with indefinite metrics, in particular, Krein spaces. But the much more wider class of strict plus-operaters is an open area for investigations. We hope that our new results on "similarity" of some subclass of strict plus-operators, namely the set of all strict plus-operators A, for which the so-called "lus-characteristic" $D(A)$ is non-negative operator, to the subclass of bistrict plus-operators, will allow to develop the theory of strict plus-operators and to obtain new results on generated linear fractional relations.

We consider applications of our results to the well-known Krein–Phillips problem of invariant subspaces of special type for sets of plus-operators acting in Krein spaces, to the so-called Koenigs embedding problem and some other fields of the Operator Theory.

Dilating Tuples of Symmetric Matrices to Commuting Tuples of Selfadjoint Operators

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The problem of dilating, up to a constant, a tuple of symmetric matrices to a tuple of commuting selfadjoint contractive operators on Hilbert space is considered. For the purposes here, an operator T on a Hilbert space $\mathscr{H}$ dilates to an operator J on a Hilbert space $\mathscr{K}$ if $\mathscr{H}$ is a subspace of $\mathscr{K}$ and $T = PJP$. Here P is the orthogonal projection of $\mathscr{K}$ onto $\mathscr{H}$. Fix a positive integer d and let $C(r)$ denote the collection of symmetric $d \times d$ matrices of norm at most r. The optimal $\vartheta(d)$ for which there exists a Hilbert space $\mathscr{K}$ and a family $\mathscr{F}$ of commuting selfadjoint contractions on $\mathscr{K}$ such that the collection $C(\vartheta(d))$ simultaneously dilates to $\mathscr{F}$ is identified via an explicit construction.

The talk is based on joint work with Bill Helton, Scott McCullough and Markus Schweighofer.

References

[1] J. W. Helton, I. Klep, S. McCullough, and M. Schweighofer, Dilations, linear matrix inequalities, the matrix cube problem and beta distributions. *Preprint*, `http://arxiv.org/abs/1412.1481`.

Matrix Convex Sets, Spectrahedrops and a Positivstellensatz

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We discuss a free noncommutative analog of real algebraic geometry – the study of polynomial inequalities and equations over the reals – focusing on matrix convex sets C and their projections $\check{C}$. Given a polynomial p, a free basic semialgebraic set $D_p = \{X : \ p(X) \geq 0\}$ which is convex can be represented as the solution set of a Linear Matrix Inequality (LMI), suggesting that convex free semialgebraic sets are rare. Further, Tarski's transfer principle fails in the free setting: The projection of a free convex semialgebraic set need not be free semialgebraic.

In this talk I will present the construction of *a sequence $C^{(d)}$ of LMI domains in larger and larger spaces whose projections $\check{C}^{(d)}$ close down on the convex hull of D_p.* It is based on free analogs of moments and Hankel matrices. Such an approximation scheme is likely the best that can be done in general. As evidence, we study the nonconvex free TV screen $p = 1 - x^2 - y^4$.

The talk is based on joint work with Bill Helton and Scott McCullough.

References

[1] J. W. Helton, I. Klep, and S. McCullough, Matrix convex hulls of free semialgebraic sets. *Trans. Amer. Math. Soc.* (to appear); `http://arxiv.org/abs/1311.5286`.

[2] J.W. Helton, I. Klep, and S. McCullough, The tracial Hahn-Banach Theorem, polar duals, matrix convex sets, and projections of free spectrahedra. *Preprint*, `http://arxiv.org/abs/1407.8198`.

Proper Maps between Free Convex Domains

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The talk will cover aspects of inequalities for free noncommutative functions in free algebras. The focus in on convexity. At this point we have classifications of free convex rational functions and free convex semialgebraic sets. There are shockingly few; all are identified with Linear Matrix Inequalities (LMIs). Now we undertake to develop a theory of change of variables to achieve free convexity. Our approach uses free algebra versions of the classical real algebraic geometry Positivstellensätze and Nullstellensätze which have been developing over the last few years.

The talk is based on joint work with Bill Helton and Scott McCullough.

About Asymptotic Behavior of Solutions for a Class of Linear Differential Equations

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The talk is devoted to presentation of method of receiving of the main term of asymptotic formulae of fundamental system of solutions at infinity for a class of scalar linear differential equations of arbitrary order with non-smooth coefficients. It is interesting that conditions on coefficients of equations such that received asymptotic formulae are true even for some classes of differential equations with coefficients–distributions. The effectiveness of these asymptotic formulae is such that they can be used for determination of deficiency indices and character of spectrum of minimal closed symmetric operators generated by corresponding differential expressions. In particular, analogue of S. A. Orlov's theorem (see [1]) about deficiency indices of linear differential operators is received for mentioned classes of differential operators proved by Orlov for a case of real-valued analytic coefficients.

The talk is based on joint work with professor K. A. Mirzoev.

Acknowledgement

Work was supported by the Russian Foundation for Basic Research (project 14-01-31136) and Russian Science Support Foundation.

References

[1] S. A. Orlov, On the deficiency index of linear differential operators. (Russian) *Doklady Akad. Nauk SSSR (N.S.)* **92** (1953), 483–486.

Three-Point Nevanlinna–Pick Problem in the Polydisc

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It is elementary to see that functions interpolating an extremal two-point Pick interpolation problem on the polydisc are just left inverses to complex geodesics.

We shall show that the same property holds for the three-point Pick problem on polydiscs, i.e. it may be expressed in terms of three-complex geodesics. Using this idea we shall be able to solve that problem. In particular we shall obtain uniqueness varieties and we shall show that extremal interpolating functions are unique only for 2-dimensional problems. As a bu product of our consideratins we shall determine a class of rational inner functions interpolating that problem and depeneding on at most three variables (for any polydic).

Possible extensions and further investigations are also discussed.

References

[1] J. Agler and J. E. McCarthy, Nevanlinna–Pick interpolation on the bidisk. *J. Reine Angew. Math.* **506** (1999), 191–204.

[2] J. Agler and J. E. McCarthy, The three point Pick problem on the bidisk. *New York J. Math.* **6** (2000), 227–236 (electronic).

[3] J. Agler and J. E. McCarthy, Distinguished varieties. *Acta Math.* **194** (2005), No. 2, 133–153.

[4] K. Guo, H. Huang, and K. Wang, Retracts in polydisk and analytic varieties with the H^∞-extension property. *J. Geom. Anal.* **18** (2008), No. 1, 148–171.

[5] Ł. Kosiński, Three-point Nevanlinna Pick problem in the polydisc. http://arxiv.org/abs/1503.03091; *J. Geom. Anal.* (submitted).

[6] Ł. Kosiński and W. Zwonek, Nevanlinna-Pick problem and uniqueness of left inverses in convex domains, symmetrized bidisc and tetrablock. *J. Geom. Anal.* (to appear); http://arxiv.org/abs/1303.0482.

[7] Ł. Kosiński and W. Zwonek, Extremal holomorphic maps in special classes of domains. http://arxiv.org/abs/1401.1657; *Ann. Sc. Norm. Super. Pisa* (to appear).

Darboux Transform of Generalized Jacobi Matrices

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Let $\mathfrak{J}$ be a monic generalized Jacobi matrix, i.e. a three-diagonal block matrix of special form, introduced by M. Derevyagin and V. Derkach in 2004. We find conditions for a monic generalized Jacobi matrix $\mathfrak{J}$ to admit a factorization $\mathfrak{J} = \mathfrak{L}\mathfrak{U}$ with $\mathfrak{L}$ and $\mathfrak{U}$ being lower and upper triangular two-diagonal block matrices of special form. In this case the Darboux transformation of $\mathfrak{J}$ defined by $\mathfrak{J}^{(p)} = \mathfrak{U}\mathfrak{L}$ is shown to be also a monic generalized Jacobi matrix. Analogues of Christoffel formulas for polynomials of the first and the second kind, corresponding to the Darboux transformation $\mathfrak{J}^{(p)}$ are found.

Some results was published (see I. Kovalyov, Darboux transformation of generalized Jacobi matrices. *Methods of Funct. Anal. and Topology* **20** (2014), No. 4, 301–320.)

Topology Optimization for Helmholtz Problems in 2d

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We consider both the forward and inverse Helmholtz problems in respect to a variable geometric object put in a bounded test domain. It has numerous applications for testing methodologies by scattering with acoustic, elastic, and electromagnetic waves.

The inverse Helmholtz problem of identification of an unknown geometric object belongs to the field of shape and topology optimization, as well as parameter estimation. The classic methods of analysis, available for the Helmholtz problem, are based mostly on the potential operator theory which is well established in case of unbounded domains.

Recently, the concept of topological derivatives was adapted to this field. The reason is that a trial object of finite size in comparison to infinitesimal one implies variation of topology of the test domain.

Based on the singular perturbation of the forward Helmholtz problem, a topology optimization approach, which is a direct one, is described for the inverse problem of object identification from boundary measurements.

Relying on the 2d setting in a bounded domain, the high-order asymptotic result is proved rigorously for the Neumann, Dirichlet, and Robin type conditions stated at the object boundary. In particular, this implies the first-order asymptotic term called a topological derivative.

For identifying arbitrary test objects, a variable parameter of the surface impedance is successful. The necessary optimality condition of minimum of the objective function with respect to trial geometric variables is discussed and realized for finding the center of the test object.

Acknowledgments

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References

[1] V. A. Kovtunenko and K. Kunisch, High precision identification of an object: optimality-conditions-based concept of imaging. *SIAM J. Control Optim.* **52** (2014), No. 1, 773–796.

The Moment Problem in Infinitely Many Variables

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The multivariate moment problem is investigated in the general context of the polynomial algebra $\mathbb{R}[x_i \mid i \in \Omega]$ in an arbitrary number of variables x_i, $i \in \Omega$. The results obtained are sharpest when the index set Ω is countable. Extensions of Haviland's theorem [1] and Nussbaum's theorem [3] are proved. Lasserre's description of the support of the measure in terms of the non-negativity of the linear functional on a quadratic module in [2] is shown to remain valid in this more general situation. Joint work with Mehdi Ghasemi and Murray Marshall. The talk is dedicated to the memory of my friend, collaborator and colleague Murray Marshall.

References

[1] E. K. haviland, On the momentum problem for distribution functions in more than one dimension. II. *Amer. J. Math.* **58** (1936), No. 1, 164–168.

[2] J. B. Lasserre, The $\mathbf{K}$-moment problem for continuous linear functionals. *Trans. Amer. Math. Soc.* **365** (2013), No. 5, 2489–2504.

[3] A. E. Nussbaum, Quasi-analytic vectors. *Ark. Mat.* **6** (1965), 179–191.

On Matrix D-Stability and Related Properties

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In this talk, we consider the positive stability and D-stability of P-matrices. We also study the property of D_θ-stability, i.e. when a matrix remains positive stable when multiplied by a positive diagonal matrix from a special class defined by a given permutation θ. We apply the obtained results to some problems connected with structured matrices. Namely, we study M-matrices and their rank one perturbations.

Positivity of Multi-Linear Maps to Detect Multi-Partite Entanglement

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Positive linear maps between matrix algebras play roles of entanglement witnesses for bi-partite cases, through the duality between tensor products and mapping spaces. We define various notions of positivity for multi-linear maps in matrix algebras including just positivity and complete positivity. They will be witnesses for various kinds of multi-partite entanglement. This talk will be based on the following two preprints:

References

[1] S.-H. Kye, Three-qubit entanglement witnesses with the full spanning properties. *J. Phys. A: Math. Theor.* **48** (2015), 235303 (arXiv:1501.00768).

[2] K. H. Han and S.-H. Kye, Various notions of positivity for bi-linear maps and applications to tri-partite entanglement. `http://arxiv.org/abs/1503.05995`.

A Functional Analytic Approach to Homogenization Problems

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This talk is devoted to the homogenization of boundary value problems in a periodically perforated domain by an approach which is alternative to those of asymptotic analysis and of classical homogenization theory.

The domain has a periodic structure, and the size of each cell is determined by a positive parameter d. The size of each periodic perforation is instead determined by a positive parameter r.

We analyze the behavior of a family of solutions of a boundary value problem defined in the periodically perforated domain as d and r degenerate to zero.

On the Jordan Structure of Holomorphic Matrices

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Let $D \subseteq \mathbb{C}$ be a domain, $L(n, \mathbb{C})$ the space of complex $n \times n$ matrices, and $A : D \to L(n, \mathbb{C})$ a holomorphic map.

A point $z_0 \in D$ will be called **exceptional of first type** if, for each neighborhood U of z_0, there exists a point $z \in U$ such that $A(z)$ has more eigenvalues than $A(z_0)$ (not counting multiplicities).

A point $z_0 \in D$ will be called **exceptional of second type** if, for each neighborhood U of z_0, there exists a point $z \in U$ such that the numbers of Jordan blocks of $A(z)$ and $A(z_0)$ are different.

It is classical that the exceptional points of first type form a discrete and closed subset of D, i.e., they are the zeros of a certain analytic function defined on D. A remarkable result of H. Baumgärtel from the 1960s is that the same is true for the exceptional points of second type.

In the talk, we outline for both results a new more elementary proof. This proof shows that the results can be complemented. For example: If A is bounded, then also the exceptional points of first (second) type are the zeros of a certain bounded holomorphic function defined on D. If A admits a continuous extension to a part of the boundary of D, then also the exceptional points of first (second) type are the zeros of a holomorphic function defined on D which admits a continuous extension to the same part of the boundary.

The method of proof also applies to matrices depending holomorphically on several complex variables. In particular, the results obtained by H. Baumgärtel in 1973 for holomorphic functions of several complex variables are obtained again.

Classification of Hermitian Operators Characteristic Polynomials with their Polynomial Invariants

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A method for classification Hermitian operators in n-dimensional unitary space (n-th order Hermitian matrices) in terms of the polynomial invariants of the operators respect to the unitary group $U(n)$ is proposed. The method enables to establish whether two Hermitian operators (two $n \times n$ Hermitian matrices) are unitary similar. The proposed set of the operators invariants contains complete information on their eigenvalues as well as on the corresponding multiplicities, and separates the classes of equivalence – orbits of operators respect to the unitary group $U(n)$. The explicit formula for the multiplicity of given eigenvalue is derived, inverse power sums of the multiplicities and the minimal polynomial of the Hermitian operator (of the $n \times n$ Hermitian matrix) are expressed in the terms of the proposed invariants too. The method proposed is algorithmic and enables realization using symbolic computations.

We have considered some examples which are important for quantum computing, namely, a separation of n-th order Hermitian matrices orbits under $U(n)$ group action, $n \leqslant 6$.

On the Inverse Problem for Discontinuous Sturm–Liouville Operator

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Consider the inverse problem of scattering theory in $[0, \infty)$

$$-y'' + q(x)y = \lambda^2 \rho(x)y, \tag{1}$$

$$y'(0) - a\lambda y(0) = 0, \tag{2}$$

where λ is a complex parameter, a is a real number, $\rho(x)$ is a positive piecewise-constant function with a finite number of points of discontinuity, the potential $q(x)$ is a real valued function and

$$\int_0^\infty (1+x)|q(x)|\,dx < \infty. \tag{3}$$

In the case $\rho(x) \equiv 1$, inverse scattering problem (1)–(3) on the half line was completely solved in [1–3] and etc. On the half line inverse problems for classical Sturm–Liouville equations with the nonlinear spectral parameter in the boundary condition were investigated in [4–6]. As different from previous works, the boundary value problem (1)–(3) have complex eigenvalues and the main equation of inverse problem is different from the classical Marchenko equation.

References

[1] V. A. Marchenko, Sturm-Liouville operators and applications. Translated from the Russian by A. Iacob. Operator Theory: Advances and Applications, 22. *Birkhäuser Verlag, Basel*, 1986.

[2] B. M. Levitan, Inverse Sturm–Liouville Problems. *VNU Science Press B. Utrecht*, 1999.

[3] V. E. Lyantse, An analog of the inverse problem of scattering theory for a nonselfadjoint operator. *Mat. Sb.* **72** (1967), 537–557.

[4] E. A. Pocheykina-Fedotova, On the inverse problem of boundary problem for second order differential equation on the half line. *Izvestiya Vuzov* **17** (1972), 75–84.

[5] V. A. Yurko, On the reconstruction of the pencils of differential operators on the half-line. (Russian) *Mat. Zametki* **67** (2000), No. 2, 316–320; translation in *Math. Notes* **67** (2000), No. 1-2, 261–265.

[6] Kh. R. Mamedov, On the inverse problem for Sturm–Liouville operator with a nonlinear spectral parameter in the boundary condition. *J. Korean Math. Soc.* **46** (2009), No. 6, 1243–1254.

On a Metric for Pseudo-Optimal Location of Cities in Pseudo-Geometrical Traveling Salesman Problem

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We use the terminology and notation of [1]. It is evident, then restoring coordinates of cities on the basis of the distance matrix, the calculated coordinates of each city depend on the selected sequence of previous cities.

Let be the set of all the possible solutions for the given problem instance of the pseudo-geometrical version of the traveling salesman problem. With the rotation by angle φ and parallel transport by the vector $\vec{a}$, we can approximate considered pseudo-alignment to another one not changing the distance matrix. For each pair $(\varphi, \vec{a})$, let us calculate the distance for the pseudo-alignments $x^i = \{(x^i_1, y^i_1), \dots, (x^i_n, y^i_n)\}$ and $x^j = \{(x^j_1, y^j_1), \dots, (x^j_n, y^j_n)\}$ using formula

$$r_{ij} = \sum_{k=1}^{n} \sqrt{(x^i_k - x^j_k)^2 + (y^i_k - y^j_k)^2}\,.$$

Let us consider all possible pairs $(\varphi, \vec{a})$ and for calculated values r_{ij} select the minimal one. Thus, for each two pseudo-alignments x^i and x^j we can consider function

$$d(x^i, x^j) = \min_{(\varphi,\vec{a})} r_{ij} = \min_{(\varphi,\vec{a})} \left\{ \sum_{k=1}^{n} \sqrt{(x^i_k - x^j_k)^2 + (y^i_k - y^j_k)^2} \right\}.$$

This function satisfies the axioms for metrics.

For proving the triangle inequality, consider inequalities

$$\sqrt{(x^i_k - x^j_k)^2 + (y^i_k - y^j_k)^2} \leqslant \sqrt{(x^i_k - x^l_k)^2 + (y^i_k - y^l_k)^2} + \sqrt{(x^l_k - x^j_k)^2 + (y^l_k - y^j_k)^2}\,,$$

where $k = 1, \dots, n$.

Denoting $x_k^i - x_k^l = a_1$, $y_k^i - y_k^l = a_2$, $x_k^l - x_k^j = b_1$, $y_k^l - y_k^j = b_2$, we obtain

$$\sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} \leqslant \sqrt{a_1^2 + a_2^2} + \sqrt{b_1^2 + b_2^2}\,,$$

which follows from Cauchy–Schwarz inequality

$$\sqrt{\sum_{k=1}^{n} (a_k + b_k)^2} \leqslant \sqrt{\sum_{k=1}^{n} a_k^2} + \sqrt{\sum_{k=1}^{n} b_k^2}\,.$$

References

[1] S. B. Makarkin, B. F. Melnikov, and M. A. Trenina, Using problem oriented metrics for solving pseudo-euclidian travelling salesman problem with Euclidian algorithms. (Russian) *Stochastic Optimization in Informatics* **10** (2014), No. 1, 63–71; `http://www.math.spbu.ru/user/gran/optstoch.htm`

About Deficiency Index of the Vector Sturm–Liouville Operator

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Let $R_+ := [0, +\infty)$ and let square matrix-functions P, Q and R of order n $(n \in \mathscr{N})$ defined on R_+ be such that $P(x)$ is non degenerate matrix, $P(x)$ and $Q(x)$ – hermitian matrices for $x \in R_+$, elements of matrix-function P^{-1}, Q and R are measurable on R_+ and summable on each its closed finite subinterval. These conditions let us define quasi-derivatives $f^{[0]}$, $f^{[1]}$ and $f^{[2]}$ of given locally absolutely continuous vector-function $f(x) = (f_1(x), f_2(x), \ldots, f_n(x))^t$ (t – symbol of transposition), assuming

$$f^{[0]} := f, \quad f^{[1]} := P(f' - Rf), \quad f^{[2]} := (f^{[1]})' + R^* f^{[1]} - Qf,$$

where * – symbol of adjoint matrix, $f^{[1]} \in AC_{loc}(R_+)$ and vector symmetric (formally self-adjoint) quasi-differential expression, assuming

$$l[f] := -f^{[2]} = -(P(f' - Rf))' - R^* P(f' - Rf) + Qf. \tag{1}$$

In the certain way the expression l defines minimal closed symmetric operator L_0 in the Hilbert space $\mathscr{L}_n^2(R_+)$. By symbol (n_+, n_-) denote deficiency index of this operator ($n \leqslant n_+, n_- \leqslant 2n$). By analogy with scalar case the operator L_0 is named by operator of Sturm–Liouville. The main goal of this paper is studying of deficiency index of operator L_0. In particular, the following theorem holds.

Theorem 1. *For the operator L_0 the limit-circle case does not realize if and only if for some sequence of nonintersecting intervals $(a_k, b_k) \subset R_+$, $k = 1, 2, \ldots$, the following condition holds*

$$\sum_{k=1}^{+\infty} \left(\int_{a_k}^{b_k} dx \int_{a_k}^{x} \|K(x,t)\|^2 \, dt \right)^{1/2} = \infty,$$

where $K(x,t)$ – Cauchy function of equation $l[f] = 0$, $||\cdot||$ - self-adjoint matrix norm.

The received results, in particular, are used for studying of differential operators, generated by expression (all derivatives are understanding in the sense of the theory of distributions):

$$l[f] = -f'' + \sum_{k=1}^{+\infty} \mathscr{A}_k \delta(x - x_k) f,$$

and generalized Jacobi matrices connected with these operators.

Acknowledgements

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Spectral and Pseudospectral Functions of Hamiltonian Systems: Development of the Results by Arov-Dym and Sakhnovich

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The main object of the talk is a Hamiltonian system $Jy' - B(t)y = \lambda H(t)y$ defined on an interval $[a, b)$ with the regular endpoint a. We define a pseudospectral function of a singular system as a matrix-valued distribution function such that the generalized Fourier transform is a partial isometry with the minimally possible kernel. Moreover, we parameterize all spectral and pseudospectral functions of a given system by means of a Nevanlinna boundary parameter. The obtained results develop the results by Arov-Dym [1] and Sakhnovich [2] in this direction.

The talk is based on the paper [3].

References

[1] D. Z. Arov and H. Dym, Bitangential direct and inverse problems for systems of integral and differential equations. Encyclopedia of Mathematics and its Applications, 145. *Cambridge University Press, Cambridge*, 2012.

[2] A. L. Sakhnovich, L. A. Sakhnovich, and I. Ya. Roitberg, Inverse problems and nonlinear evolution equations. Solutions, Darboux matrices and Weyl–Titchmarsh functions. De Gruyter Studies in Mathematics, 47. *De Gruyter, Berlin*, 2013.

[3] V. I. Mogilevskii, Spectral and pseudospectral functions of Hamiltonian systems: development of the results by Arov–Dym and Sakhnovich. *Methods Funct. Anal. Topol.* **21** (2015) (accepted).

Lie Derivations on a Certain Class of Algebras

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Let A be a unital algebra. Each Lie derivation $L : A \longrightarrow A$ is called proper whenever L is of the form $d + \tau$ where d is a derivation on A and $\tau : A \longrightarrow Z(A)$ is a linear map. In this talk, we investigate this property for these algebras, triangular algebras, generalized matrix algebras, full matrix algebras and $B(X)$.

References

[1] D. Benkovič, Generalized Lie derivations on triangular algebras. *Linear Algebra Appl.* **434** (2011), No. 6, 1532–1544.

[2] W.-S. Cheung, Lie derivations of triangular algebras. *Linear Multilinear Algebra* **51** (2003), No. 3, 299–310.

[3] Y. Du and Y. Wang, Lie derivations of generalized matrix algebras. *Linear Algebra Appl.* **437** (2012), No. 11, 2719–2726.

[4] W. Jing and F. Lu, Lie derivable mappings on prime rings. *Linear Multilinear Algebra* **60** (2012), No. 2, 167–180.

Motion Equations of Deformable Bodies of Hencky's Isotropic Hyperelastic Material

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It is shown, that Hencky's isotropic hyperelastic material model, based on the logarithmic strain tensor $\mathbf{E}^{(0)}$, and Noll stress tensor $\bar{\boldsymbol{\tau}}$, pair of conjugate Lagrangian tensors, prolongate Hooke's law from area of infinitesimal deformations to area of moderate deformations. New representation of the fourth-order elasticity tensor $\mathbb{C}$ for Hencky's hyperelastic isotropic material is obtained. This fourth-order elasticity tensor realize linear connection between material rates of symmetric Lagrangian second Piola–Kirchhoff stress tensor $\mathbf{S}^{(2)}$ and Green–Lagrange strain tensor $\mathbf{E}^{(2)}$ through eigenvalues and eigenprojections of right Cauchy–Green strain tensor $\mathbf{C}$.

It is shown, that obtained elasticity tensor possesses both minor symmetries, and the major symmetry. This properties have major theoretical meaning in incremental formulations of hyperelasticity, when formulate variational principles for equations, stated with velocities, and criteria of uniqueness and stability of the solutions for this equations.

It is well known, that performing a finite element analysis (FEA) on a hyperelastic material is difficult due to nonlinearity, large deformation, and material instability. FEA software package MSC.Marc is utilized for the simulations, based on different hyperplastic material models. In this paper Lagrangian formulation of Hencky's isotropic hyperelastic material constitutive relations is implemented into MSC.Marc code. Reliability of implementation proves to be true due to the comparison of numerical solutions obtained with the use of MSC.Marc code with exact solutions of three-dimensional problems on simple shear and on uniaxial extension of a rod with Henckys isotropic hyperelastic material model. New solutions of a problem on origin of a neck and postcritical deformation of the rod are obtained at its extension by the prescribed displacement of the edge face.

References

[1] S. N. Korobeynikov, Objective tensor rates and applications in formulation of hyperelastic relations. *J. Elasticity* **93** (2008), No. 2, 105–140.

[2] S. N. Korobeynikov, Families of continuous spin tensors and applications in continuum mechanics. *Acta Mech.* **216** (2011), No. 1-4, 301-332.

[3] S. N. Korobeynikov and A. A. Oleinikov, Lagrangian formulation of Hencky's hyperelastic material. (Russian) *Dalnevost. Mat. Zh.* **11** (2011), No. 2, 155–180.

[4] S. N. Korobeynikov, A. A. Oleinikov, A. Yu. Larichkin, A. V. Babichev, and V. V. Alyokhin, Computer implementation of Lagrangian formulation of Hencky's isotropic hyperelastic material constitutive relations. (Russian) *Dalnevost. Mat. Zh.* **13** (2013), No. 2, 222–249.

Variational Formulation of Wing Panel Forming

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It is shown that inverse problems of creep forming of wing panel can be represented in a quasistatic variational principles. It is required to determine the deflections or loads that must be applied to the plate blank to obtain the specified residual deflections at the time of removal of the external loads and fastening. The problem of optimal forming is formulated as follows: it is required to find a strain path element of the blank, so that at the time the specified creep strains are obtained and the damage parameter is minimal.

Steady-state values of the obtained functionals corresponding to the solutions of the problems of inelastic strain and springback are determined by applying a finite element procedure to the functionals. The modeling results can be used to calculate the die tooling, determine the panel processibility, and control panel rejection in the course of forming.

The paper gives basics of the developed computer-aided system of design, modeling, and digital manufacture of wing integral panels. System application data resulting from computation of 3D-involute of a CAD-based panel model, determination of working surfaces of die tooling, three-dimensional analysis of stresses, and simulation of panel shaping under diverse thermomechanical and speed conditions are demonstrated.

References

[1] A. I. Oleinikov, Integrated design of wing panel manufacture processes. *Key Engineering Materials* **554-557** (2013), 2175–2186.

[2] A. I. Oleinikov and K. S. Bormotin, The inverse problems of wing panel manufacture processes. *AIP Conf. Proc.* **1564** (2013), 1048-1051.

On Spectrum of Banach Couples

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We introduce the notion of spectrum for a Banach couple which relates to joint critical values of the two norms involved. However the definition of spectrum has to be equivalent norms robust.

We use the standard definitions and notation from [1], [2] in what follows.

Let $\{X_0, X_1\} = \overline{X}$ be a Banach couple, $x \in X_0 + X_1$, $s, t > 0$. The J.Peetre K-functional is defined as

$$K(s, t, x, \overline{X}) = \inf_{x_0 \in X_0, x_1 \in X_1} s\|x_0\|_{X_0} + t\|x_1\|_1,$$

where g.l.b. is taken over all $x_0 \in X_0$ and $x_1 \in X_1$ such that $x = x_0 + x_1$.

Definition 1. An element $e \in X_0 \cap X_1$ is said to be eigenvector of the couple $\overline{X}$, if $K(1/\|e\|_0, 1/\|e\|_1, e, \overline{X}) = 1$, and the point $(\|e\|_0^{-1}, \|e\|_1^{-1})$ of the projective space $\mathbb{RP}$ is said to be eigenvalue, corresponding to the eigenvector e.

Definition 2. An element $e \in X_0 \cap X_1$ is said to be pseudo-eigenvector of $\overline{X}$, if for some $0 < C < 1$ we have

$$K(1/\|e\|_0, 1/\|e\|_1, e, \overline{X}) > C.$$

The corresponding $(\|e\|_0^{-1}, \|e\|_1^{-1}) \in \mathbb{RP}$ is said C-eigenvalue of the Banach couple $\overline{X}$. The set of all pseudo-eigenvalues with a fixed C is called a pseudo-spectrum of $\overline{X}$ and is denoted by $\sigma(\overline{X})$.

Theorem 1. *There is a constant C such that the set of C-eigenvalues is non-empty for all Banach couples.*

Let us denote by $K(\overline{X})$ the set of all functions $K(s, t, x, \overline{X})$, where x runs over $(X_0 + X_1)^\circ$.

Theorem 2. *The set $K(\overline{X})$ is contained in the set of all $\sup_{e \in E} \min(s \backslash \|e\|_0, t \backslash \|e\|_1)$ up to equivalence, where e runs over some subset E of pseudo-eigenvectors of $\overline{X}$.*

Recall that positive function $\varphi(s, t)$ of two positive variables is called interpolation function if φ increase on s and t, and is homogeneous of the degree one. Let S be a subset of $\mathbb{RP}$, generated by positive (A, B). Denote by φ_S an interpolation function which is equal to the minimal extension of the restriction of φ to the set of (A, B), corresponding to S.

Theorem 3. *Let $\overline{X}$, $\overline{Y}$ be Banach couples with pseudo-spectra $\sigma(\overline{X})$, $\sigma(\overline{Y})$, then for each bounded linear operator $T : \overline{X} \to \overline{Y}$ and each interpolation function φ we have $T : \varphi_{\sigma(\overline{X})}(X_0, X_1)_{p_0,p_1} \to \varphi_{\sigma(\overline{Y})}(Y_0, Y_1)_{p_0,p_1}$, where $\varphi(X_0, X_1)_{p_0,p_1}$ is the generalized Lions–Peetre interpolation construction for $0 < p_0, p_1 \leqslant \infty$.*

This theorem leads to serious improvement in classical interpolation theorems applied to couples with different pseudo-spectra.

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References

[1] J. Bergh and J. Löfström, Interpolation spaces. An introduction. Grundlehren der Mathematischen Wissenschaften, No. 223. *Springer-Verlag, Berlin–New York*, 1976.

[2] V. I. Ovchinnikov, Interpolation functions and the Lions–Peetre interpolation construction. (Russian) *Usp. Mat. Nauk* **69** (2014), No. 4, 103–168; translation in *Russ. Math. Surv.* **69** (2014), No. 4, 681–741; doi:10.1070/RM2014v069n04ABEH004908.

Scattering of Vortices in Abelian Higgs Models on Riemann Surfaces

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Abelian Higgs models on Riemann surfaces are generalizations of the well-known $(2+1)$-dimensional Abelian Higgs model on the plane which arises in the theory of superconductivity. Many properties of models on Riemann surfaces are analogous to properties of the model on the plane.

If we fix the so-called topological charge in the model on the plane (it is an integer number N) and suppose that $N > 0$ then all the static solutions of the model appear to be minimizers of the potential energy which solve some first-order equations (so-called vortex equations; their solutions with the charge N are called N-vortices). Up to gauge transformations all the static solutions are parametrized by their zeroes and the number of zeroes (with multiplicities) is always equal to N. So the moduli space of N-vortices is the N-th symmetric power $S^N\mathbb{R}^2$.

The kinetic energy functional in the model on the plane defines a Riemannian metric on moduli spaces of N-vortices for any $N \in \mathbb{N}$ (kinetic metric). Manton in 1982 suggested so-called adiabatic principle which states that geodesics of kinetic metric on moduli space of N-vortices are good approximations to "slow" dynamical solutions with topological charge

N. This principle was justified for Abelian Higgs model on the plane only recently. Study of geodesics allows to obtain results on vortex motion. For example one can show that after symmetric head-on collision of N vortices they scatter on the angle π/N (if $N = 2$ we obtain right-angle scattering).

In the models on compact Rimannian surfaces static solutions are also parametrized (up to gauge equivalence) by their zeroes, but the number of zeroes is given by the Chern number of the corresponding line bundle over the surface. The kinetic energy functional again gives a metric on moduli spaces of static solutions. The construction of the metric is the main topic of the talk.

We suggest that the adiabatic principle also holds true in models on Riemann surfaces. Using geodesic approximation one can obtain results on vortex motion in these models that are similar to results for the plane.

Transformations of Matrix Structures Work Again

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Matrices with the structures of Toeplitz, Hankel, Vandermonde and Cauchy types are omnipresent in modern computations in Sciences, Engineering, and Signal and Image Processing. These four matrix classes have distinct features, but in our paper in Mathematics of Computation 1990 we showed that Vandermonde and Hankel multipliers can be applied to transform each class into the others, and then we demonstrated how by using these transforms we can readily extend any successful matrix inversion algorithm from one of these classes to all the others. The power of this approach was widely recognized later, when novel numerically stable algorithms solved nonsingular Toeplitz linear systems of equations in quadratic (versus classical cubic) arithmetic time based on transforming Toeplitz into Cauchy matrix structures. More recent papers combined the same transformation with a link of the Cauchy matrices to the Hierarchical Semiseparable matrix structure, which is a specialization of matrix representations employed by the Fast Multipole Method. This produced numerically stable algorithms that approximated the solution of a nonsingular Toeplitz linear system of equations in nearly linear arithmetic time. We first revisit the successful method of structure transformation, covering it comprehensively. Then we analyze the latter approximation algorithms for Toeplitz linear systems and extend them to approximate multiplication of Vandermonde and Cauchy matrices by a vector and to approximate solution of Vandermonde and Cauchy linear systems of equations provided that they are nonsingular and well-conditioned. We decrease the arithmetic cost of the known numerical approximation algorithms for these tasks from quadratic to nearly linear, and similarly for the computations with the matrices having structures that generalize the structures of Vandermonde and Cauchy matrices and for polynomial and rational evaluation and interpolation. We also accelerate a little further the known numerical approximation algorithms for a nonsingular Toeplitz or Toeplitz-like linear system by employing distinct transformations of matrix structures, and we briefly comment on some natural research challenges, particularly some promising applications of our techniques to high precision computations.

Spaces of Smooth Vectors – Alternate Characterizations

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We provide a short review of various characterizations of some spaces of rapidly decreasing functions: the Schwartz space, Gelfand–Shilov (GS) spaces and generalized Gelfand–Shilov–Roumieu (GSR) spaces. We emphasize that part of these characterization theorems can be obtained from more general theorems on smooth vectors for Lie groups representations related to families of Hermitian or skew-Hermitian operators in a separable Hilbert space. The commutation relations between these operators play a central role. A special attention is payed to the Heisenberg group. Our approach was inspired by ter Elst's paper [1].

Generalized GS or GSR spaces can be also defined by using, instead of sequences of positive numbers, their associated functions. We provide a characterization of those weight functions which may be used for the definition of GSR spaces.

References

1 A. F. M. ter Elst, Gevrey spaces and their intersections. *J. Austral. Math. Soc. Ser. A* **54** (1993), No. 2, 263–286.

2 M. Pascu, On the definition of Gelfand–Shilov spaces. *Ann. Univ. Buchar. Math. Ser.* **1(LIX)** (2010), No. 1, 125–133.

3 M. Pascu, On the characterization of Gelfand–Shilov–Roumieu spaces. *J. Operator Theory* **72** (2014), No. 2, 475–485.

On the Restricted Maximal Operator of Féjer Means with Respect to Vilenkin–Fourier Series

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Weisz [3] proved that maximal operator of Fejér means

$$\sigma^* := \sup_{n \in \mathbb{N}} |\sigma_n f|$$

with respect to Vilenkin systems, is bounded from the Hardy space $H_{1/2}$ to the space weak-$L_{1/2}$. In [1] it was proved that there exist a martingale $f \in H_p$ $(0 < p \leqslant 1/2)$ such that

$$\sup_n \|\sigma_n f\|_p = +\infty.$$

On the other hand, Weisz [4] proved that the maximal operator

$$\sigma^{\#} f = \sup_{n \in \mathbb{N}} |\sigma_{2^n} f|$$

with respect to Walsh system (Walsh system is one of the simplest example of Vilinkin systems), is bounded from the martingale Hardy space H_p to the space L_p for $p > 0$.

This lecture is devoted to review and characterize a maximal subspace Q of natural numbers $\mathbb{N}$, such that the restricted maximal operator

$$\sigma^{\#,*} f = \sup_{n_k \in Q \subset \mathbb{N}} |\sigma_{n_k} f|$$

is bounded from the martingale Hardy spaces H_p to the space L_p for $0 < p \leqslant 1/2$, from which as a consequence, theorem of Weisz [4] can be obtained.

The talk is based on joint work with co-authors Lars-Erik Persson (see L. E. Persson, G. Tephnadze [2]).

References

[1] G. Tephnadze, Fejér means of Vilenkin–Fourier series. *Studia Sci. Math. Hungar.* **49** (2012) No. 1, 79–90.

[2] L. E. Persson and G. Tephnadze: A sharp boundedness result concerning some maximal operators of Vilenkin–Fejér means. *Mediter. Math. J.* (to appear).

[3] F. Weisz, Weak type inequalities for the Walsh and bounded Ciesielski systems. *Anal. Math.* **30** (2004), No. 2, 147–160.

[4] F. Weisz, Summability of multi-dimensional Fourier series and Hardy spaces. Mathematics and its Applications, 541. *Kluwer Academic Publishers, Dordrecht*, 2002.

The Effective Conductivity Properties in Temperature Dependent Doubly Periodic 2D Composite Materials

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We consider 2D unbounded doubly periodic composite materials with circular non-overlapping inclusions when the matrix and inclusions are occupied by temperature sensitive materials. The corresponding non-linear boundary value problem is solved in analytic form in the case when the conductivity coefficients of the matrix and the composite constituencies are proportional. Moreover, we find the effective conductivity tensor in explicit form and compare the outcomes with a few results from literature

The talk is based on joint works [1, 2] with co-authors David Kapanadze and Gennady Mishuris.

References

[1] D. Kapanadze, G. Mishuris, and E. Pesetskaya, Improved algorithm for analytical solution of the heat conduction problem in doubly periodic 2D composite materials. *Complex Var. Elliptic Equ.* **60** (2015), No. 1, 1–23.

[2] D. Kapanadze, G. Mishuris and E. Pesetskaya, Exact solution of a nonlinear heat conduction problem in a doubly periodic 2D composite material. *Arch. Mech.* **67** (2015), No. 2, 157–178.

Relationship Between the Equality $A^{*n}A^n = (A^*A)^n$ and Composition Operators in L^2 Spaces

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It is proved that a closed densely defined operator A is quasinormali if and only if the equality $A^{*n}A^n = (A^*A)^n$ holds for $n = 2, 3$. Let W be bounded injective weighted shift which satisfies the equality $W^{*n}W^n = (W^*W)^n$. We prove that operator W is then quasinormal. We will construct examples of bounded, non-quasinormal operator A which satisfies equality $A^{*n}A^n = (A^*A)^n$. An example of such a operator is given in the class of weighted shifts on directed trees. What is important, the directed tree used in the construction is rootless and therefore the operator in example is unitarily equivalent to a composition operator in L^2-space.

Rate of Decay of the Bernstein Numbers

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Let X, Y be Banach spaces and let $\mathscr{L}(X, Y)$ be the space of all bounded linear operators from X to Y. An operator $T \in \mathscr{L}(X, Y)$ is called *superstrictly singular* (*SSS* for short) if there are no number $c > 0$ and no sequence of subspaces $E_n \subset X$, $\dim E_n = n$, such that

$$\|Tx\| \geqslant c\|x\| \quad \text{for all} \quad x \quad \text{in} \quad \cup_n E_n\,. \tag{1}$$

Put for an operator T

$$b_n(T) = \sup \min_{x \in S_E} \|Tx\|\,,$$

where supremum is taken over all n-dimensional subspaces $E \subset X$ and S_E is the unit sphere of E. Evidently, T is SSS if and only if $b_n(T) \to 0$ as $n \to \infty$ and the greatest constant c for which (1) is satisfied, is equal to $\lim\limits_{n\to\infty} b_n(T)$. Every compact operator is SSS and T has finite rank if and only if $b_n(T) = 0$ beginning with some integer n. The $b_n(T)$, which are called the *Bernstein numbers*, were considered in Approximation and Operator Theory. The constants $b_n(T)$ show how small is the T-image of the unit sphere S_X. For a compact operator T in a Hilbert space H they coincide with s-*numbers* which are defined as eigenvalues of the operator $(T^*T)^{1/2}$. We present two results on Bernstein numbers.

1. If a Banach space X contains uniformly complemented ℓ_2^n's then there exists a universal constant $b = b(X) > 0$ such that for each Banach space Y, and any sequence $d_n \downarrow 0$ there is a bounded linear operator $T : X \to Y$ with the Bernstein numbers $b_n(T)$ of T satisfying $b^{-1}d_n \leqslant b_n(T) \leqslant bd_n$ for all n.

2. Let $\mathscr{P}$ be the Pisier space [1] and H be a Hilbert space. There is $\lambda > 0$ such that for every operator $T \in \mathscr{L}(\mathscr{P}, H)$ and every n

$$b_n(T) \leqslant \frac{\lambda}{\sqrt{n}} \|T\|.$$

References

[1] G. Pisier, Counterexamples to a conjecture of Grothendieck. *Acta Math.* **151** (1983), No. 3-4, 181–208.

Birkhoff–James Approximate Orthogonality Sets

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The *numerical range* of a square matrix $A \in \mathbb{C}^{n\times n}$ is the compact and convex set $F(A) = \{x^*Ax \in \mathbb{C} : \ x \in \mathbb{C}^n, \ x^*x = 1\}$, and it has been studied extensively for many decades and is useful in studying matrices and operators. Stampfli and Williams (1968) observed that $F(A)$ can be written

$$F(A) = \big\{\mu \in \mathbb{C} : \ \|A - \lambda I_n\|_2 \geqslant |\mu - \lambda|, \ \forall\, \lambda \in \mathbb{C}\big\} = \\ = \bigcap_{\lambda\in\mathbb{C}} \big\{\mu \in \mathbb{C} : \ |\mu - \lambda| \leqslant \|A - \lambda I_n\|_2\big\}.$$

Hence, $F(A)$ is an infinite intersection of closed (circular) disks $\mathscr{D}(\lambda, \|A - \lambda I_n\|_2) = \{\mu \in \mathbb{C} : \ |\mu - \lambda| \leqslant \|A - \lambda I_n\|_2\}$ ($\lambda \in \mathbb{C}$). Inspirited by this intersection property, Chorianopoulos, Karanasios and Psarrakos (2009) introduced a definition of numerical range for rectangular complex matrices. In particular, for any $A, B \in \mathbb{C}^{n\times m}$ with $B \neq 0$, and any matrix norm $\|\cdot\|$, the *numerical range of A with respect to B* is defined as

$$F_{\|\cdot\|}(A; B) = \big\{\mu \in \mathbb{C} : \ \|A - \lambda B\| \geqslant |\mu - \lambda|, \ \forall\, \lambda \in \mathbb{C}\big\} = \\ = \bigcap_{\lambda\in\mathbb{C}} \mathscr{D}\big(\lambda, \|A - \lambda B\|\big).$$

This set is obviously compact and convex, and it satisfies basic properties of the standard numerical range. Moreover, it is nonempty if and only if $\|B\| \geqslant 1$.

In this work, we introduce a new range of values for rectangular matrices and matrix polynomials, which is based on the notion of Birkhoff–James approximate orthogonality and generalizes the numerical range $F_{\|\cdot\|}(A;B)$. We also show that it is quite rich in structure by establishing some of its main properties, we study the boundary points and the case of matrix polynomials, and we give illustrative examples to verify our results.

This is a joint work with Christos Chorianopoulos.

On the Hyperreflexivity of Subspaces of Toeplitz Operators on Regions in the Complex Plane

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The existence of invariant subspace for the bounded operator on the Hilbert space can be equivalently put as existence of the rank one operator in the preannihilator of the algebra generated by the operator. The reflexivity of an algebra of operators (or a subspace of operators) means that there are so many rank one operators in the preannihilator of the algebra (subspace) of operators that they determine the algebra (subspace) itself. The hyperreflexivity of algebra (subspace) of operators means that the usual distance from any operator to the algebra (subspace) can be controlled by the distance given by rank one operators in the preannihilator of the algebra (subspace) of operators. Changing rank one to rank k operators we get k-hyperreflexivity.

It will be shown that the algebra of analytic Toeplitz operators on the Hardy spaces on Jordan regions in the complex plane or the upper half-plane is hyperreflexive, the subspace of all Toeplitz operators on these Hardy spaces is 2-hyperreflexive and we get 2-hyperreflexivity of any weak* closed subspace of all Toeplitz operators on these Hardy spaces.

Joint work with W. Młocek.

Asymptotic Behaviour of Eigenvalues of Hankel Operators

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I will discuss compact Hankel operators realized in $\ell^2(\mathbb{Z}_+)$ as infinite matrices Γ with matrix elements $h(j+k)$. The focus of the talk is the asymptotic behaviour of the singular values $s_n(\Gamma)$ of these Hankel operators as $n \to \infty$. First, I will discuss the following naive

Conjecture: for $\alpha > 0$,

$$h(j) = O(j^{-1}(\log j)^{-\alpha}), \ \ j \to \infty \Longrightarrow s_n(\Gamma) = O(n^{-\alpha}), \ \ n \to \infty.$$

I will explain that this conjecture is essentially true (it is true for $0 < \alpha < 1/2$ and it becomes true for $\alpha \geqslant 1/2$, if we impose some additional regularity assumptions on the sequence h).

Next, I will discuss the corresponding asymptotic result: suppose

$$h(j) = j^{-1}(\log j)^{-\alpha}(1 + o(1)) \ \text{ as } \ j \to \infty;$$

then the singular values of Γ satisfy $s_n(\Gamma) = c(\alpha)n^{-\alpha}$ as $n \to \infty$, where the asymptotic coefficient $c(\alpha)$ can be explicitly computed. Some information on the eigenvalues of Γ is also available.

If time permits, I will also discuss similar results for Hankel operators $\mathbf{\Gamma}$ realized in $L^2(\mathbb{R}_+)$ as integral operators with kernels $\mathbf{h}(t + s)$.

This is joint work with Dmitri Yafaev (University of Rennes 1).

Perturbations of Discontinuous Functions of Self-Adjoint Operators

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Let A, B be self-adjoint operators, and let φ be a continuous function on the real line which tends to zero at infinity. It is well known that if $A - B$ is compact, then $\varphi(A) - \varphi(B)$ is also compact. Thus, the essential spectrum of the latter operator is $\{0\}$. In the talk, I will address the following question:

What can be said about the spectrum of $\varphi(A) - \varphi(B)$ if the function φ has a jump discontinuity?

It is not difficult to see that if φ has a jump, then $\varphi(A) - \varphi(B)$ can acquire non-trivial essential spectrum. It turns out that in some situations this spectrum can be described in much detail. This can be done in the scattering theory framework. One has to assume that the pair of operators A, B satisfies some standard conditions of scattering theory. Then the spectrum of $\varphi(A) - \varphi(B)$ can be given as a union of some explicit spectral bands; the description of these bands involves the spectrum of the scattering matrix for the pair A, B.

I will discuss this result in the case when A, B are the Schrödinger operators in $L^2(\mathbb{R}^d)$:

$$A = -\Delta, \quad B = -\Delta + V,$$

where V is a short-range potential.

I will also discuss a related result which describes the spectrum of $\varphi_\varepsilon(A) - \varphi_\varepsilon(B)$, where φ_ε is a smooth function which approaches a limiting function φ_0 as $\varepsilon \to 0$. If φ_0 has a jump, then the eigenvalues of $\varphi_\varepsilon(A) - \varphi_\varepsilon(B)$ concentrate to the spectral bands of the limiting operator $\varphi_0(A) - \varphi_0(B)$. I will describe the rate of this concentration and the corresponding spectral density.

Commuting Toeplitz Operators and their Geometry

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Let D be a bounded symmetric domain whose group of isometries has a connected component given by the simple Lie group G. It is well know that there is a unitary representation of G on $\mathscr{A}^2_\lambda(D)$, where the latter denotes a weighted Bergman space of holomorphic square-integrable functions on D for a suitably weighted Lebesgue measure. A similar setup can be considered for D an open set of a projective space where homogeneous coordinates are defined where now G is the group of isometries for the Fubini–Study metric.

On the other hand, for $a \in L^\infty(D)$ the Toeplitz operator on $\mathscr{A}^2_\lambda(D)$ is defined by

$$T^{(\lambda)}_a(f) = B^{(\lambda)}(af)$$

where $B^{(\lambda)}$ is the orthogonal projection $L^2_\lambda(D) \to \mathscr{A}^2_\lambda(D)$. With respect to these operators, a very interesting problem is to determine sets of symbols whose Toeplitz operators generate commutative algebras, either C^* or just Banach.

We will present some results showing that most (maybe all) of the examples of such commutative algebras have some Riemannian and symplectic geometry that can be associated through their symbols and that appear naturally from the G-action. In fact, this has been a source of motivation to continue the construction of such algebras.

Integral Equations of Diffraction Problems with Unbounded Smooth Obstacles

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The talk is devoted to the boundary integral equations method for the diffraction problems on obstacles D in $\mathbb{R}^n$ with smooth unbounded boundaries for Helmholtz operators with variable coefficients

$$\mathscr{H}u(x) = \big(\rho(x)\nabla \cdot \rho^{-1}(x)\nabla + a(x)\big)u(x), \quad x \in \mathbb{R}^n,$$

where ρ, a belong to the space of the infinitely differentiable functions on $\mathbb{R}^n$ bounded with all derivatives. We introduce the single and double layer potentials associated with the operator $\mathscr{H}$, and reduce by means of these potentials the Dirichlet, Neumann, and Robin problems to pseudodifferential equations on the infinite boundary ∂D.

Applying the limit operators method we study the Fredholm properties and the invertibility of the boundary pseudodifferential operators in the Sobolev spaces $H^s(\partial D)$, $s \in \mathbb{R}$.

References

[1] V. S. Rabinovich, Integral equations of diffraction problems with unbounded smooth obstacles. *Integral Equations Oper. Theory*, 2015 (to appear).

Multiplication Form of Normal Operator on a Quaternionic Hilbert Space

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In this article we prove the Multiplication form of a normal operator on a quaternionic Hilbert space: Let T be a right linear bounded normal operator on a quaternionic Hilbert space H. We prove that there exists σ- additive measure space (Ω, μ), a unitary operator $U\colon H \to L^2(\Omega; \mathbb{H}; \mu)$ and an essentially bounded function $f\colon \Omega \to \mathbb{C}$ such that

$$T = U^* M_f U,$$

where M_f is the multiplication operator on $L^2(\Omega; \mathbb{H}; \mu)$ induced by f. As a consequence of this result we also prove the integral representation of a normal operator.

References

[1] J. B. Conway, A course in functional analysis. Graduate Texts in Mathematics, 96. *Springer-Verlag, New York*, 1985.

[2] R. Ghiloni, V. Moretti, and A. Perotti, Continuous slice functional calculus in quaternionic Hilbert spaces. *Rev. Math. Phys.* **25** (2013), no. 4, 1350006, 83 pp.

[3] K. Viswanath, Normal operations on quaternionic Hilbert spaces. *Trans. Amer. Math. Soc.* **162** (1971), 337–350.

A Distance Function in Reproducing Kernel Hilbert Spaces

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Associated with a reproducing kernel Hilbert space H on a set X is a metric on X which encodes some properties of functions in the space. If H has a complete Pick kernel then this metric agrees with the Gleason–Harnack metric on the spectrum of the maximal ideal space of the multiplier algebra. It also agrees with the pseudohyperbolic metric induced from the associated embedding of X into the complex ball.

First- and Second-Order Sensitivity Analysis for a Body with a thin Rigid Inclusion

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We deal with the two-dimensional problem of the elastic equilibrium of a body containing a thin rigid inclusion. The rigid inclusion is considered to be soldered into elastic medium; at the external boundary the body is fixed and it is in equilibrium under the action of volume forces. The problem is formulated in a variational form, i.e., in the form of minimization of the energy functional on the set of admissible displacements. The first variation and the second variation of the solution with respect to the shape of the domain are calculated.

Shape sensitivity analysis plays an important role in optimization problems of the shape of elastic bodies [1, 2]. Many works are devoted to the first-order sensitivity analysis, which makes it possible to write out necessary conditions of optimality. At the same time knowing the second-order shape derivative of a cost functional makes it possible to obtain a sufficient condition of optimality and improve numerical methods for shape optimization problems. We use perturbation techniques to derive the first- and second-order material derivatives of the solution (for example, see [3]). The main difficulty in investigation of such problems concerning rigid inclusions is that the set of admissible displacements of the perturbed and unperturbed problems can not be mapped each other (for example, see [4, 5]). Moreover, it implies that the corresponding material derivative does not belong to the set of admissible displacements of the initial problem.

As an example of using the obtained results, the problem of seeking an optimal shape of rigid inclusion is considered. Necessary and sufficient conditions of the local minimum are written out.

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References

[1] J. Haslinger and R. A. E. Mäkinen, Introduction to shape optimization. Theory, approximation, and computation. Advances in Design and Control, 7. *Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA*, 2003.

[2] A. M. Khludnev and J. Sokolowski, Modelling and control in solid mechanics. International Series of Numerical Mathematics, 122. *Birkhäuser Verlag, Basel*, 1997.

[3] V. A. Kovtunenko, Sensitivity of cracks in 2D-Lamé problem via material derivatives. *Z. Angew. Math. Phys.* **52** (2001), No. 6, 1071–1087.

[4] E. M. Rudoy, The Griffith formula and Cherepanov–Rice integral for a plate with a rigid inclusion and a crack. *J. Math. Sci. (N. Y.)* **186** (2012), No. 3, 511–529.

[5] P. I. Plotnikov and E. M. Rudoy, Shape sensitivity analysis of energy integrals for bodies with rigid inclusions and cracks. (Russian) *Dokl. Akad. Nauk* **440** (2011), No. 5, 589–592; translation in *Dokl. Math.* **84** (2011), No. 2, 681–684.

Parallel Computational Algorithms in Problems of the Dynamics of Structurally Inhomogeneous Media

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Under modeling the processes of propagation of the stress waves in geomaterials (granular and porous media, soils and rocks) it is necessary to take into account their structural inhomogeneity. To make possible the description of deformation of materials with different resistance to tension and compression, the rheological method was supplemented by a new element, a rigid contact, which serves for imitation of a perfectly granular material with rigid particles [1]. By using rigid contact in combination with conventional rheological elements (a spring simulating elastic properties of a material, a viscous damper, and a plastic hinge) one can construct constitutive equations of granular materials and soils with elastic-plastic particles and of porous materials like metal foams. Rotational motion of particles in the material microstructure can be considered within the framework of mathematical model of the Cosserat continuum.

In order to obtain correct numerical solutions for structurally inhomogeneous materials by means of finite-difference methods, computations must be performed on a grid whose meshes are smaller than the characteristic size of the particles of a material. To solve 2D and 3D dynamic problems, parallel algorithms and programs are worked out, which allow to distribute the computational load between a lot of cluster nodes. Parallel program systems *2Dyn_Granular* and *3Dyn_Granular* are intended for numerical realization of the universal mathematical model describing small strains of elastic, plastic and granular materials. In the case of an elastic material this model is reduced to the system of equations, hyperbolic by Friedrichs, written in terms of velocities and stresses in a symmetric form. In the case of an elastic-plastic material the model is a special formulation of the Prandtl–Reuss theory in the form of variational inequality with one-sided constraints on stresses. Generalization of the model to describe the deformation of a granular material is obtained by means of the rheological approach, taking into account different resistance of a material to tension and compression. Program systems *2Dyn_Cosserat* and *3Dyn_Cosserat* allow to solve dynamic problems of the Cosserat elasticity theory, taking into account rotations of the particles of microstructure of a material. This model is formulated as the system of equations, hyperbolic by Friedrichs, written in terms of velocities of translational and rotational motion, as well as stresses and couple stresses. Some computations, demonstrating the efficiency of numerical algorithms and programs, were performed on clusters.

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References

[1] O. Sadovskaya and V. Sadovskii, Mathematical modeling in mechanics of granular materials. With a foreword by Holm Altenbach. Advanced Structured Materials, 21. *Springer, Heidelberg*, 2012.

New Mathematical Models of Structurally Inhomogeneous Media, Having Different Resistance to Tension and Compression

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Almost all known natural and synthetic materials resist tensile and compressive deformations in different ways. For some metals and alloys the mismatch of elastic modules, yield strengths, creep diagrams and stress relaxation curves, obtained in experiments in tension and compression, is so small that it may be neglected. However, in the study of alternating signs strain in granular and porous media such neglect is not possible. Under the construction of constitutive equations for such materials an original method is proposed, in which the non-differentiable convex potentials – indicator functions of special cones in spaces of strain and stress tensors – are used along with the classical potentials of elastic, viscous and plastic media to describe the asymmetry of mechanical properties with respect to tension and compression [1]. In this method the tensor constitutive relationships for simple materials are formulated in the form of subdifferential inclusions with dual potentials of stresses and strains. For materials with complex spatial rheology, constitutive relationships are obtained by combining the relationships of simple media from consideration of rheological schemes of uniaxial deformation.

Closed mathematical models of the dynamics of elastic-plastic and granular media are formulated as variational inequalities for hyperbolic operators with one-sided constraints, describing the transition of a material into plastic state. On this basis integral a priori estimates are constructed in the characteristic cones of operators, which imply the uniqueness and continuous dependence on initial data of solutions of the Cauchy problem and of the boundary value problems with dissipative boundary conditions. Original algorithms of shock-capturing type are developed, which can be considered as a realization of splitting method with respect to physical processes, automatically satisfying the properties of monotonicity and dissipativity on discreet level and applicable for computation of the solutions with singularities of strong discontinuities (elastic-plastic shock waves) and discontinuities of displacements. For numerical solution of 2D and 3D dynamic problems the two-cyclic splitting method with respect to spatial variables is applied.

Earlier thermodynamically self-consistent systems of conservation laws were considered by S. K. Godunov and his colleagues, applied to the models of reversible thermodynamics (the theory of elasticity, fluid dynamics and electrodynamics). The present work is devoted to a generalization of this approach for the analysis of thermodynamically irreversible models taking into account plastic deformation of a medium.

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This work was supported by the Russian Foundation for Basic Research (Grant # 14-01-00130).

References

[1] O. Sadovskaya and V. Sadovskii, Mathematical modeling in mechanics of granular materials. With a foreword by Holm Altenbach. Advanced Structured Materials, 21. *Springer, Heidelberg*, 2012.

Toeplitz Operators with Quasi-Homogeneuos Quasi-Radial Symbols on Some Weakly Pseudoconvex Domains

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On the weakly pseudo-convex domains Ω_p^n we introduce quasi-homogeneous quasi-radial symbols. These are used to prove the existence of a commutative Banach algebra of Toeplitz operators on Bergman space of Ω_p^n. We also show that group theoretic and geometric properties for our symbols are satisfied. The results presented here contain the geometric description of the symbols introduced by N. Vasilevski in [2] for the unit ball $\mathbb{B}^n$.

The talk is based on joint work with Raúul Quiroga-Barranco.

References

[1] R. Quiroga-Barranco and A. Sáanchez-Nungaray, Toeplitz operators with quasi-homogeneuos quasi-radial symbols on some weakly pseudoconvex domains. *Complex Anal. Oper. Theory* **9** (2015), No. 5, 1111–1134; doi: 10.1007/s11785-014-0407-x.

[2] N. Vasilevski, Quasi-radial quasi-homogeneous symbols and commutative Banach algebras of Toeplitz operators. *Integral Equations Operator Theory* **66** (2010), No. 1, 141–152.

Invariant and Wandering Subspaces of Reproducing Kernel Hilbert Spaces

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We discuss briefly recent and not so recent results about invariant subspaces (Beurling–Lax–Halmos type theorem) and analytic models and operator positivity of bounded linear operators on Hilbert spaces. We also discuss some related results on wandering subspaces in both one and several variables.

Part of this talk is based on joint work with Monojit Bhattacharjee and Dinesh K. Keshari.

Quotient Modules of $H^2(\mathbb{D}^n)$

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Let $H^2(\mathbb{D}^n)$ be the Hardy space over the unit polydisc $\mathbb{D}^n$ and $\mathcal{S}$ and $\mathcal{Q}$ be a pair of closed subspaces of $H^2(\mathbb{D}^n)$. Then $\mathcal{Q}$ ($\mathcal{S}$) is said to be a quotient module (submodule) of the Hardy module $H^2(\mathbb{D}^n)$ if $\mathcal{Q}$ ($\mathcal{S}$) is a joint $(M_{z_1}^*, \ldots, M_{z_n}^*)$-invariant ($(M_{z_1}, \ldots, M_{z_n})$-invariant) subspace of $H^2(\mathbb{D}^n)$.

Starting with a brief review of the Beruling theorem concerning submodules and quotient modules of $H^2(\mathbb{D})$, we consider a class of quotient modules of $H^2(\mathbb{D}^n)$ ($n \geqslant 2$), namely, doubly commuting quotient modules of $H^2(\mathbb{D}^n)$. In particular, we prove that a quotient module of $H^2(\mathbb{D}^n)$ is doubly commuting if and only if it is an "elementary tensor product" of one variable quotient modules. Also, by studying the associated submodules of such quotient modules, we shall explain that for a class of submodules of $H^2(\mathbb{D}^n)$, $n \geqslant 2$, essential normality is a 2 variables phenomenon. We shall conclude indicating a few trends and perspectives of Rudin type submodules including rigidity type results of submodules of $H^2(\mathbb{D}^n)$, $n \geqslant 2$.

Real Fibered Morphisms

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Hyperbolic polynomials play an important role in real algebraic geometry, due to their nice convexity properties (see for example [1] and [2]). A general notion of hyperbolicity for real projective varieties was introduced in [3]. One says that a real algebraic variety $X \subset \mathbb{P}^d$ of dimension $1 \leqslant k < d$ is hyperbolic with respect to a real linear subspace $V \subset \mathbb{P}^d$ of dimension $d - k - 1$ if for every real linear subspace $V \subset U \subset \mathbb{P}^d$ of dimension $d - k$, we have that $U \cap X \subset X(\mathbb{R})$, where $X(\mathbb{R})$ stands for the real points of X. It is then straightforward to see that the projection from V induces a finite surjective morphism $f \colon X \to \mathbb{P}^k$ that satisfies $f(x)$ is real if and only if x is real. This leads us to define the following notion:

Definition. A morphism of two real quasi-projective varieties $f \colon X \to Y$ is called real fibered if f is finite, surjective, flat and it satisfies $f(x) \in Y(\mathbb{R})$ if and only if $x \in X(\mathbb{R})$. We will say that a real projective variety X is weakly hyperbolic if it admits a real fibered morphism to $\mathbb{P}^k$.

In this talk we will discuss real fibered morphisms. We will show that for curves weak hyperbolicity is equivalent to the fact that the curve admits a hyperbolic embedding. We will then show that a real fibered morphism is necessarily unramified at real points. Then we will discuss some applications of this theory. Finally, time permitting we will describe relation of this notion to Ulrich sheaves and determinantal representations.

The talk is based on joint work with Mario Kummer.

References

[1] H. H. Bauschke, O. Güller, A. S. Lewis, and H. S. Sendov, Hyperbolic polynomials and convex analysis. *Canad. J. Math.* **53** (2001), No. 3, 470–488.

[2] L. Gårding, An inequality for hyperbolic polynomials. *J. Math. Mech.* **8** (1959), 957–965.

[3] E. Shamovich and V. Vinnikov, Livsic-type determinantal representations and hyperbolicity. `http://arxiv.org/abs/1410.2826`.

On the Continuity of the Spectral Factorization Mapping: Quantitative Results

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It is well known that the matrix spectral factorization mapping is continuous from $L^1(\mathbb{T})$ to $H^2(\mathbb{T})$ under the additional assumption of uniform integrability of the logarithms of the spectral densities to be factorized (G. Janashia, E. Lagvilava, and L. Ephremidze; S. Barclay). The talk will report on a joint project with Lasha Epremidze and Ilya Spitkovsky, which aims at obtaining quantitative results characterizing this continuity. Although we have obtained some satisfactory estimates in the scalar setting, much remains to be done for the general matrix case.

Statistical Approximation Properties of Modified Durrmeyer q-Baskakov Type Operators with Two Parameter α and β

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The main aim of this study is to obtain statistical approximation properties of these operators with the help of the Korovkin type statistical approximation theorem. Rates of statistical convergence by means of the modulus of continuity and the Lipschitz type maximal function are also established. Our results show that rates of convergence of our operators are at least as fast as classical Durrmeyer type modified Baskakov operators.

References

[1] V. N. Mishra and P. Patel, The Durrmeyer type modification of the q-Baskakov type operators with two parameter α and β. *Numer. Algorithms* **67** (2014), No. 4, 753–769.

[2 A. Aral and V. Gupta, On the Durrmeyer type modification of the q-Baskakov type operators. *Nonlinear Anal.* **72** (2010), No. 3-4, 1171–1180.

[3] A. Aral and V. Gupta, Generalized q-Baskakov operators. *Math. Slovaca* **61** (2011), No. 4, 619–634.

[4] D.D. Stancu, Approximation of functions by means of a new generalized Bernstein operator. *Calcolo* **20** (1983), No. 2, 211–229 (1984).

[5] G. Gasper and M. Rahman, Basic hypergeometric series. With a foreword by Richard Askey. Encyclopedia of Mathematics and its Applications, 35. *Cambridge University Press, Cambridge*, 1990.

[6] V. G. Kac and P. Cheung, Quantum calculus. Universitext. *Springer-Verlag, New York*, 2002.

[7] V. N. Mishraa, P. Sharma, A. Kliçman, and D. Jain, Statistical approximation properties of Stancu type q-Baskakov-Kantorovich operators. *Filomat, University of Niš, Faculty of Science and Mathematics, Department of Mathematics, Niš*, 2015 (accepter).

Griffith Formula and J-Integral for Elastic Bodies with Timoshenko Inclusions

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The talk is concerned with a two-dimensional free boundary problem for an elastic body containing a thin elastic inclusion. The thin elastic inclusion is modeled within the framework of Timoshenko beam theory. Partial delamination of the inclusion results in appearing a crack. The Signorini conditions for mutual nonpenetration of the crack faces are fulfilled. The problem was studied by Itou and Khludnev [1], who used a variational inequality for modeling the phenomenon. We derive the Griffith-type formula for the first derivative of the energy functional with respect to the crack length. It is proved that the formula for the derivative can be represented as a path independent integral along a curve surrounding the crack tip. The invariant integral consists of regular and singular terms and is an analogue of the classical Eshelby–Cherepanov–Rice J-integral. The results obtained are based on techniques developed by Destuynder and Djaoua [2] for traction free stress faces and extended by Khludnev and Sokolowski [3] to the case of mutual nonpenetration conditions on the cracks faces.

Acknowledgments

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References

[1] H. Itou and A. M. Khludnev, On delaminated thin Timoshenko inclusions inside elastic bodies. *Math. Methods Appl. Sci.*, 2014; doi: 10.1002/mma.3279.

[2] P. Destuynder and M. Djaoua: Sur une interprétation mathématique de l'intégrale de Rice en théorie de la rupture fragile. (French) *Math. Methods Appl. Sci.* **3** (1981), No. 1, 70–87.

[3] A. M. Khludnev and J. Sokolowski, Griffith formulae for elasticity systems with unilateral conditions in domains with cracks. *Eur. J. Mech. A Solids* **19** (2000), No. 1, 105–119.

Estimate for Powers of Norm Operator, Generated by Rotation

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In this talk we discuss the estimation for the norm operator $\|B^n\|$, where B is an operator generated by rational rotation. It is known Gelfand formula of the spectral radius of operator

$$R(B) = \lim_{n \longrightarrow \infty} \|B^n\|^{1/n}.$$

The previous equality is fulfilled for different behavior of $\|B^n\|$: if $\|B^n\| = R(B)^n$; if $\|B^n\| = C_0 R(B)^n$ and if $\|B^n\| = e^{\sqrt{n}} R(B)^n$. Therefore, arise a question obtaining more information on the behavior of $\|B^n\|$.

We consider the weighted shift operators generated by rational rotation acting by the following formula

$$Bu(t) = a(t)u(t+h),$$

where $a(t)$ is a given function with period 1 and h is a rational number.

The estimates

$$R(B)^n \leqslant \|B^n\| \leqslant C_0 R(B)^n,$$

where C_0 ia a constant, are obtained.

References

[1] A. Antonevich, Linear functional equations. Operator approach. Translated from the 1988 Russian original by Victor Muzafarov and Andrei Iacob. Operator Theory: Advances and Applications, 83. *Birkhäuser Verlag, Basel*, 1996.

[2] A. B. Antonevich and J. Makowska, On spectral properties of weighted shift operators generated by mappings with saddle points. *Complex Anal. Oper. Theory* **2** (2008), No. 2, 215–240.

[3] O. Nevanlinna, On the growth of the resolvent operators for power bounded operators. *Linear operators (Warsaw, 1994)*, 247–264, Banach Center Publ., 38, *Polish Acad. Sci., Warsaw*, 1997.

Hysteresis Operators as a constitutive Relations in Modeling of Irreversible Processes of Polarization of the Ferroelectric Materials

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The hysteresis operators, arising in a problems of polarization and depolarization of polycrystalline ferroelectric materials, were built at the form of equations in differentials. In the three dimension case, they can be reduced to a system of quasi-linear partial differential equations of the first order [1]. If considered only an uniaxial application of the electric field, that is the important case from a practical point of view, we obtain a system of ordinary differential equations to determine the unknown values as a hysteresis dependencies between the polarization vector and the vector of the applied electric field. In the one-dimensional case was obtained the explicit form of the limiting polarization in the form well known distribution of Langevin.

The numerical methods were proposed for solving of the obtained equations. To solve systems of ordinary differential equations was been used a Runge–Kutta method of 4-th order. Additionally for the solutions of the equations in differentials we have used methods of successive approximation, allowing to determine the increments of the unknown functions after several iterations. We carried out a numerous numerical experiments and built a large and small dielectric hysteresis loops. It was found that by changing the parameters of the model, we can change the behavior of the hysteresis curves, changing their shape and slope. It was found that parameters of the model is possible to dispose such a way that the calculated hysteresis curves not only qualitatively but also quantitatively coincided with the experimental data.

The research results can be used in the construction of constitutive relations for modeling a nonlinear and irreversible processes of polarization of polycrystalline ferroelectric materials by the finite element method, which is widely used for solution such problems.

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References

[1] A. V. Belokon and A. S. Skaliukh, Mathematical modeling of irreversible processes of polarization. (Russian) *FIZMATLIT, Moscow*, 2010.

Unbounded Subnormal Weighted Composition Operators in L^2-Spaces

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The aim of my talk is to present a selection of recent results on unbounded weighted composition operators in L^2-spaces. The most interesting and difficult ones are related to the question of subnormality. The main invention in this matter is introducing the so-called consistency condition (and its "inverse" version as well) which helped us to establish a criterion for subnormality of unbounded weighted composition operators in L^2-spaces, which is new even in the bounded case. The subnormality of composition operators in L^2-spaces has been studied in [1]. The class of weighted composition operators in L^2-spaces includes multiplication operators, composition and partial composition operators in L^2-spaces as well as weighted shifts on countably infinite directed trees.

The talk is based on a joint work with the co-authors P. Budzyński, Z. J. Jabłoński, and I. B. Jung (see [2] for its preliminary version).

References

[1] P. Budzyński, Z. J. Jabłoński, I. B. Jung, and J. Stochel, Unbounded subnormal composition operators in L^2-spaces. *J. Funct. Anal.*, 2015; doi:10.1016/j.jfa.2015.01.024.

[2] P. Budzyński, Z. J. Jabłoński, I. B. Jung, and J. Stochel, Unbounded weighted composition operators in L^2-spaces. `http://arxiv.org/abs/1310.3542`.

A Solvability Criterion for Certain Conformal Differential Operators on Euclidean Space

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In his talk we are concerned with certain partial differential operators of the first order, conformally invariant under the action of the conformal group of the Euclidean space $\mathbb{R}^n$. They were introduced in a seminal paper of Stein and Weiss [1] and later investigated by many authors, e.g. [2–4]. These operators are directly related to representations of the spherical principal series for the conformal group $\mathbf{SO}(n+1,1)$ of the Euclidean space $\mathbb{R}^n$.

In this note we give a solvability criterion for those operators and show that it involves a certain singular integral operator of the Calderon–Zygmund type.

References

[1] E. Stein and G. Weiss, Generalization of the Cauchy–Riemann equations and representations of the rotation group. *Amer. J. Math.* **90** (1968), 163–196.

[2] B. Ørsted, Generalized gradients and Poisson transforms. *Global analysis and harmonic analysis (Marseille-Luminy, 1999)*, 235–249, Sémin. Congr., 4, *Soc. Math. France, Paris*, 2000.

[3] H. M. Reimann, Invariant systems of differential operators. T*opics in modern harmonic analysis, Vol. I, II (Turin/Milan, 1982)*, 723–768, *Ist. Naz. Alta Mat. Francesco Severi, Rome*, 1983.

[4] A. Strasburger, Differential operators of gradient type associated with spherical harmonics. *Ann. Polon. Math.* **53** (1991), No. 2, 161–183.

On Boundedness of the Hardy Operator in Morrey-Type Spaces

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We study the boundedness of the Hardy operator

$$(H_\alpha f)(x) = \frac{1}{|B(0,|x|)|^{1-\frac{\alpha}{n}}} \int\limits_{B(0,|x|)} f(y)dy, \quad x \in \mathbb{R}^n,$$

where $\alpha \in \mathbb{R}$, in local and global Morrey-type spaces $LM_{p\theta,w(\cdot)}$, $GM_{p\theta,w(\cdot)}$, respectively, characterized by numerical parameters $0 < p,\theta \leqslant \infty$ and a functional parameter w, a non-negative measurable function on $(0,\infty)$, the spaces of all functions $f \in L_p^{loc}(\mathbb{R}^n)$ with finite quasi-norms

$$\|f\|_{LM_{p\theta,w(\cdot)}} = \big\| w(r)\|f\|_{L_p(B(0,r))} \big\|_{L_\theta(0,\infty)},$$
$$\|f\|_{GM_{p\theta,w(\cdot)}} = \sup_{x\in\mathbb{R}^n} \big\| w(r)\|f\|_{L_p(B(x,r))} \big\|_{L_\theta(0,\infty)},$$

respectively.

This problem is reduced to the problem of a continuous embedding of one local Morrey-type space to another one. This allows obtaining, for all admissible values of the numerical parameters $\alpha, p_1, p_2, \theta_1, \theta_2$, sufficient conditions on the functional parameters w_1 and w_2 ensuring the boundedness of H_α from $LM_{p_1\theta_1,w_1(\cdot)}$ to $LM_{p_2\theta_2,w_2(\cdot)}$ and from $GM_{p_1\theta_1,w_1(\cdot)}$ to $GM_{p_2\theta_2,w_2(\cdot)}$. Moreover, for a certain range of the numerical parameters and under certain a priori assumptions on w_1 and w_2 these sufficient conditions coincide with the necessary ones.

Joint work with V. I. Burenkov and P. Jain.

Detailed exposition of the results is contained in the paper [1].

Acknowledgement

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References

[1] V. I. Burenkov, P. Jain, and T. V. Tararykova, On boundedness of the Hardy operator in Morrey-type spaces. *Eurasian Math. J.* **2** (2011), No. 1, 42–70.

Stability of Multidimensional Systems and Stein Inequalities: the Free Noncommutative Case

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For an $n \times n$ matrix A, it is well known that stability of A, in the sense that $A^k x \to 0$ as $k \to \infty$ for any vector x, holds if and only if one of the following equivalent conditions is satisfied:

(i) $I - zA$ is invertible for all z in the closed unit disk $\overline{\mathbb{D}}$;

(ii) there exists an invertible $n \times n$ matrix S such that $\|S^{-1}AS\| < 1$;

(iii) there exists a positive definite solution X to the strict Stein equation $X - A^*XA > 0$.

In the context of certain nD-systems, the equivalence between appropriately modified (structured) versions of these three conditions fails, in particular the equivalence between the modifications of (i) and (ii). However, interpreting (the modification of) (i) with z as a free noncommutative variable from an appropriate domain, it is possible to prove a generalization of the classical stability result with three equivalent statements.

The talk is based on joint work with Joe Ball and Gilbert Groenewald.

The Bezout–Corona Problem Revisited: Wiener Space Setting

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The matrix-valued Bezout-corona problem $G(z)X(z) = I_m$, $|z| < 1$, is studied in a Wiener space setting. It turns out that all Wiener solutions can be described explicitly in terms of two matrices and a square analytic Wiener function Y satisfying $\det Y(z) \neq 0$ for all $|z| \leqslant 1$. It is also shown that some of the results hold in the H^∞ setting, but not all. Nevertheless, in this case, using the two matrices and the function Y, all H^2 solutions to the Bezout-corona problem can be described explicitly in a form analogous to the one appearing in the Wiener setting.

The talk is based on joint work with G. Goenewald and M. A. Kaashoek.

On the Arazy Conjecture Concerning Schur Multipliers on Schatten Ideals

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We confirm in the affirmative the conjecture made by Arazy in 1982. Suppose that f is a continuously differentiable function on $[-1, 1]$ such that

$$|f'(t)| \leqslant C|t|^\alpha, \quad t \in [-1, 1],$$

for some constant $C > 0$ and $0 < \alpha < \infty$. Let $\{\lambda_j\}_{j=1}^\infty \in \ell^r$ for some $0 < r < \infty$. If $0 < p < \infty$ and $1 < q < \infty$ satisfy

$$\frac{1}{p} = \frac{\alpha}{r} + \frac{1}{q},$$

then the matrix

$$\left\{\frac{f(\lambda_j)-f(\lambda_k)}{\lambda_j-\lambda_k}\right\}_{j,k=1}^{\infty}$$

is a Schur multiplier from the Schatten class $\mathscr{S}^q$ into $\mathscr{S}^p$. We also consider a general version of this conjecture for arbitrary quasi-Banach ideals of compact operators.

Joint work with D. Potapov and F. Sukochev.

Maximal Volume and Expansions with Small Coefficients

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Given a system of n vectors from $\mathbb{C}^m$, we want to find a subsystem consisting of k vectors so that the expansion of any other vector over this subsystem has the coefficients sufficiently small in modulus. The maximal volume principle allows one to find a subsystem of $k = m$ vectors with a guarantee that all expansions have the coefficients in modulus bounded by 1. If we increase k, then smaller coefficients could be obtained. We present different settings of the problem and some new results and discuss applications to the problem of construction of low-rank approximations to matrices and tensors.

On Some Properties of a Weighted Space

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In this study, we define $A_{w,\vartheta}$ to be space of the intersection of $L^p_w(\mathbb{R}^n)$ and $L^{q(\cdot)}_\vartheta(\mathbb{R}^n)$ spaces. Also, we investigate some inclusions and embedding properties of the space. Moreover, we discuss other basic properties of $A_{w,\vartheta}$ space.

References

[1] A. T. Gürkanlı, Compact embeddings of the spaces $A^p_{w,\omega}(\mathbf{R}^d)$. *Taiwanese J. Math.* **12** (2008), No. 7, 1757–1767.

[2] C. Unal and İ. Aydın, Some results on a weighted convolution algebra. *Proc. Jangjeon Math. Soc.* **18** (2015), No. 1, 109–127.

[3] H. Reiter, Classical harmonic analysis and locally compact groups. *Clarendon Press, Oxford*, 1968.

[4] İ. Aydın and A. T. Gürkanli, On some properties of the spaces $A_w^{p(x)}(\mathbb{R}^n)$. *Proc. Jangjeon Math. Soc.* **12** (2009), No. 2, 141–155.

[5] O. Kováčik and J. Rákosnĵik, On spaces $L^{p(x)}$ and $W^{k,p(x)}$. *Czechoslovak Math. J.* **41(116)** (1991), No. 4, 592–618.

[6] R. H. Fischer, A. T. Gürkanli, and T. S. Liu, On a family of weighted spaces. *Math. Slovaca* **46** (1996), No. 1, 71–82.

[7] T.-sun Liu and A. van Rooij, Sums and intersections of normed linear spaces. *Math. Nachr.* **42** (1969), 29–42.

Singular Traces and Perturbation Formulae

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Let H, V be self-adjoint operators such that V belongs to the weak trace class ideal. In this talk we discuss higher order perturbation formulae

$$\tau\bigg(f(H+V)-\sum_{j=0}^{n-1}\frac{1}{j!}\,\frac{d^j}{dt^j}\,f(H+tV)\Big|_{t=0}\bigg)=\int\limits_{\mathbb{R}}f^{(n)}(t)\,dm_n(t),$$

where τ is a singular trace on the weak trace class ideal and m_n is a finite measure that is not necessarily absolutely continuous. These results extend the classical Krein and Koplienko trace formulae as well as recently established higher order trace formulae to the weak trace class ideal.

The talk is based on joint work with D. Potatov, F. Sukochev, D. Zanin.

On the Maximal Function of a Contraction Operator

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The maximal function of a contraction operator $T \in \mathscr{L}(\mathscr{H})$ arises in the factorization process of an operator valued semispectral measure, i.e. to each operator valued semispectral measure on the unit torus $\mathbb{T}$ it corresponds an outer operator valued function on the unit disc $\mathbb{D}$, having some maximal properties. In the special case of a contraction, to the attached semispectral measure of T the corresponding outer function is an L^2-bounded analytic function which was called the maximal function of T and has the form $M_T(\lambda) = D_{T^*}(I - \lambda T^*)^{-1}$, where $\lambda \in \mathbb{D}$ and D_T is the defect operator of T.

In the the particular $C_{.0}$ case, the Sz.-Nagy–Foias functional model reduces to the functional representation given by the maximal function $M_T(\lambda)$, i.e. $\mathbf{H} = M_T\mathscr{H} \subset H^2(\mathscr{D}_{T^*})$, where $(M_T h)(\lambda) = M_T(\lambda)h$. In this case M_T becomes an isometry, and the functional model for T^* is given by the restriction of the backward shift to $\mathbf{H}$, and can be expressed with the maximal function of T as $\mathbf{T}^* = \frac{1}{\lambda}[M_T(\lambda)h - M_T(0)h]$.

Analogously, the maximal function of T^* has the form $M_{T^*}(\lambda) = D_T(I - \lambda T)^{-1}$, and for the discrete linear system generated by the rotation operator $R_T = \begin{bmatrix} T & D_{T^*} \\ D_T & -T^* \end{bmatrix}$ the operators M_T and M_{T^*} become the controllability and the observability operators, respectively.

Some other properties of the maximal function are analyzed in various cases, too.

References

[1] B. Sz.-Nagy and C. Foiaş, Harmonic analysis of operators on Hilbert space. Translated from the French and revised *North-Holland Publishing Co., Amsterdam-London; American Elsevier Publishing Co., Inc., New York; Akadémiai Kiadó, Budapest*, 1970.

[2] I. Valuşescu, The maximal function of a contraction. *Acta Sci. Math. (Szeged)* **42** (1980), No. 1-2, 183–188.

[3] I. Valuşescu, Some connections between the maximal function and linear systems. *Math. Rep. (Bucur.)* **12(62)** (2010), No. 2, 189–199.

Tensors of Hankel and Loewner-Type

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Given a sequence of numbers h_i, $i = 0, 1, 2, \ldots$, the corresponding Hankel matrix H is defined as the following structured matrix

$$H = \begin{bmatrix} h_0 & h_1 & h_2 & \cdots \\ h_1 & h_2 & & \\ h_2 & & \ddots & \\ \vdots & & & \end{bmatrix}.$$

Given four sequences of numbers x_i, y_i, f_i, g_i, $i = 1, 2, 3, \ldots$, where all the numbers x_i and y_i are different from each other, the corresponding Loewner matrix L is defined as

$$L = \left[\frac{f_i - g_j}{x_i - y_j}\right]_{i=1,2,\ldots}^{j=1,2,\ldots}.$$

These two classes of structured matrices are strongly connected to rational interpolation problems.

In this talk, we will look at some possible generalizations of Hankel and Loewner matrices to tensors. The use of these tensors of Hankel and Loewner-type to solve certain interpolation and approximation problems will also be studied.

Matrix-Valued Hermitian Positivstellensatz, Lurking Contractions, and Contractive Determinantal Representations of Stable Polynomials

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Let $\mathbf{P}$ be a direct sum of matrix-valued polynomials $\mathbf{P}_i$ in d variables over $\mathbb{C}$. It is clear that if a polynomial p admits a determinantal representation

$$p = \det(I_{|n|} - K\mathbf{P}(z)_n),$$

where $\mathbf{P}(z)_n = \bigoplus(\mathbf{P}_i(z) \otimes I_{n_i})$ for some $n = (n_1, \ldots, n_d) \in \mathbb{Z}_+^d$, $|n| = n_1 + \cdots + n_d$, and $K \in \mathbb{C}^{|n| \times |n|}$ is a contraction, then p has no zeroes on the domain $\mathscr{D}_{\mathbf{P}} = \{z \in \mathbb{C}^d\colon \ \|\mathbf{P}(z)\| < 1\}$. We prove a weak version of the converse statement: if a polynomial p has no zeroes on $\overline{\mathscr{D}_{\mathbf{P}}}$, then (subject to an appropriate Archimidean condition so that in particular $\mathscr{D}_{\mathbf{P}}$ is bounded) there exists a polynonmial q so that the product pq has a contractive determinantal representation as above.

The main ingredients of the proof are a matrix-valued Hermitian Positivstellensatz, a lurking contraction argument, and a finite-dimensional contractive realization theorem for (matrix-valued) rational functions that are regular on $\overline{\mathscr{D}_{\mathbf{P}}}$ and whose associated Agler norm is strictly less than 1.

This is a joint work with Anatoly Grinshpan, Dmitry Kaliuzhnyi-Verbovetskyi, and Hugo Woerdeman. In a related talk of D. Kaliuzhnyi-Verbovetskyi in the special section on Free Noncommutative Analysis and Its Applications, the finite-dimensional contractive realization theorem will be used together with noncommutative techniques to deduce a much stronger statement in the case when the polynomially defined domain $\mathscr{D}_{\mathbf{P}}$ is a product of matrix balls: in this case no auxilliary factor q is needed. In the special case of the polydisc these results are related, via the Cayley transform and the interplay between complex and real stable polynomials, to the generalized Lax conjecture in convex algebraic geometry which states that every hyperbolicity cone is spectrahedral (equivalently, every hyperbolic polynomial has a positive definite selfadjoint determinantal representation, possibly with an auxilliary factor satisfying extra conditions).

Noncommutative Completely Positive Kernels

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Functions of free noncommuting variables were first considered by Joseph L. Taylor in his monumental work on noncommutative spectral theory in the 1970s [4, 5] and became a topic of active research in the last several years [6, 3, 1]. I will discuss the analogue, in the context of free noncommutative function theory, of the classical correspondence between positive kernels and reproducing kernel Hilbert spaces [2]. The role of positive kernels is taken over by noncommutative completely positive kernels that generalize both the usual positive kernels and completely positive maps. This talk is based on joint work with Joseph Ball and Gregory Marx, and is closely related to the plenary talk of J. Ball on the Nevanlinna–Pick interpolation problem in the free noncommutative setting.

References

[1] J. Agler and J. E. McCarthy, Global holomorphic functions in several non-commuting variables. *Canadian J. Math.* (to appear); preprint `http://arxiv.org/abs/1305.1636`.

[2] N. Aronszajn, Theory of reproducing kernels. *Trans. Amer. Math. Soc.* **68** (1950), 337–404.

[3] D. S. Kaliuzhnyi-Verbovetzkyi and V. Vinnikov, Foundations of noncommutative function theory. Math. Surveys and Monographs 199, *Amer. Math. Society*, 2014.

[4] J. L. Taylor, A general framework for a multi-operator functional calculus. *Advances in Math.* **9** (1972), 183–252.

[5] J. L. Taylor, Functions of several noncommuting variables. *Bull. Amer. Math. Soc.* **79** (1973), 1–34.

[6] D.-V. Voiculescu, Free analysis questions II: the Grassmannian completion and the series expansions at the origin. *J. Reine Angew. Math.* **645** (2010), 155–236.

Matrix Fejér–Riesz Theorem with Gaps

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Two equivalent versions of the matrix Fejér–Riesz theorem characterize positive semidefinite matrix polynomials on the complex unit circle $\mathbb{T}$ and on the real line $\mathbb{R}$. We extend the characterization to arbitrary closed basic semialgebraic sets $\mathscr{K} \subseteq \mathbb{T}$ and $K \subseteq \mathbb{R}$ by the use of matrix preorderings from real algebraic geometry. In the $\mathbb{T}$-case the characterization is the same for all sets $\mathscr{K}$, while in the $\mathbb{R}$-case the characterizations for compact and non-compact sets K are different. Furthermore, we study a complexity of the characterizations in terms of a bound on the degrees of the summands needed. We prove, for which sets $\mathscr{K}$, K the degrees can be bounded by the degree of the given matrix polynomial and provide counterexamples for the sets, where this is not possible. At the end we give an application of results to a matrix moment problem.

References

[1] A. Zalar, Matrix Fejér–Riesz theorem with gaps. `http://arxiv.org/pdf/1503.06034.pdf`.

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