

Spectral Regularity of Banach algebras and Sums of Idempotents

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A basic result from complex function theory states that the contour integral of the logarithmic derivative of a scalar analytic function can only vanish when the function has no zeros inside the contour. Question: *does this result generalize to the vector-valued case?*

We suppose the function takes values in a Banach algebra. The answer depends on which Banach algebra. Positive results have been obtained for large classes of algebras, among them that of the polynomial identity Banach algebras. To deal with the latter, one needs non-commutative Gelfand theory which involves the use of families of matrix representations.

Logarithmic residues have much to do with sums of idempotents. Pursuing this connection, negative answers to the above question have come up via the construction of non-trivial zero sums of a finite number of idempotents. It is intriguing that we need only five idempotents in all known examples. The idempotent constructions relate to deep problems concerning the geometry of Banach spaces and general topology. In particular a novel approach to the construction of Cantor type sets plays a role.

The talk is a report on joint work with Torsten Ehrhardt (Santa Cruz, California) and Bernd Silbermann (Chemnitz, Germany).